THE DEVELOPMENT OF A SYSTEMATIC PLAN FOR USING VISUAL AIDS IN THE TEACHING OF STATISTICAL QUALITY CONTROL

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by
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Approved:

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GLOSSARY

SYMBOLS USED IN THIS WORK

X	Individual values representing measurements, specimens, or items
$\overline{X}$	Average (arithmetic mean) of a subgroup (sample) of values of X
$\overline{X}'$	The absolute or theoretical average of the population of values of X
$\overline{\overline{X}}$	The average of averages, or the average of a group of $\overline{X}$ values
n	The number of specimens in a sample
N	The number of specimens in a lot to be sampled
R	The range--difference between largest and smallest items in a sample
$\overline{R}$	The average range--average of values of R from several samples
σ	The Standard deviation--"Sigma"
$\sigma_{\overline{X}}$	Standard deviation of a group of $\overline{X}$ values
σ_{np}	Standard deviation of a group of np values, etc.
UCL	Upper Control Limit
LCL	Lower Control Limit
p	Fraction defective (np/n)
p_1	Sometimes used to denote the AQL
p_2	Sometimes used to denote the LTPD
q	One minus p $(1 - p)$
$\overline{p}$	Average of fraction defective of several samples
np	The number of defectives

$n\bar{p}$	The average number of defectives in several samples
c	The number of defects
$\bar{c}$	The average number of defects in several samples
$c, c_1 c_2$	Acceptance numbers--the number of defectives in an accepted sample
AQL	Acceptable Quality Level--an agreed quality basis for acceptance
α	Producer's risk--probability of rejection of lots above AQL
LTPD	Lot Tolerance Percent Defective--minimum acceptable quality
β	Consumer's risk--probability of acceptance of lots worse than LTPD
AOQ	Average Outgoing Quality
AOQL	Average Outgoing Quality Limit

CHAPTER I

INTRODUCTION

In the growing number of books on statistical quality control, there has been only an occasional laboratory exercise based on the employment of visual aids, and these use primarily coins, dice, numbered chips, and bowls of colored beads. While it is recognized that this apparatus has been somewhat traditional as a means of making theoretical probabilities appear real to the student in elementary statistics, it is the purpose of this study to investigate many existing devices used in illustrating various phases of this instruction, and to develop one appropriate laboratory demonstration for the teaching of each of nine selected topics in elementary statistical quality control.

When quality control is taught to inspectors, operators, or plant executives, it is imperative that a minimum of time be expended on theory, and every effort be made to keep things practical. Although the applications of statistics to quality control are based on a branch of applied mathematics developed from the theory of probability, ". . . the mathematician has already condensed any involved calculations for Quality Control practice to simple tables and constants. As will be seen early in the study and use of Quality Control methods, the best procedure is to try them."[1]

In presenting an elementary course in statistical quality control, laboratory experiments can be designed to highlight the progressive presentation of control and sampling techniques with illustrative examples to prove and demonstrate the theory at each stage of training. In the writer's opinion properly selected experiments using appropriate materials and apparatus should:

(a) Create additional interest by encouraging individual participation in the demonstrations,

(b) Assist in pointing up the practical aspects of the subject by placing in the trainees' hands appropriate materials to measure, count or inspect,

(c) Teach by presenting circumstances requiring repetition of procedures or computations which help one to learn by doing,[2]

(d) Help convert the skeptic from the attitude, "It is all too impractical and probably would not work out in actual shop practice,"

(e) Help build confidence in control techniques even when using actual measurements, count, etc. made by the man himself.

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[1] Clifford W. Kennedy, Quality Control Methods (New York: Prentice-Hall, 1948), p. 6.

[2] L. C. Young, "Teaching Quality Control Theory to Engineers," Journal of Engineering Education, 32:672, April 1942.

It is recognized that in many industrial plants it is possible to obtain appropriate materials for study directly out of the inspection or production lines and to use the corresponding gages to illustrate principles of process control and acceptance sampling. This technique should be used wherever it is feasible to assist the mental transition from illustration to application. Other plants may, for reasons of convenience or expense, find it more expedient to set up special training facilities. The latter procedure is almost universally used in engineering college instruction. Illustrations used in this thesis are primarily designed to utilize common materials, reasonable in cost and readily available. The measuring devices and equipment recommended can be readily prepared or improvised if not already available in the average manufacturing plant or engineering college laboratory.

Problems which can be adapted to demonstration or individual participation vary widely in scope, in difficulty, and in value, as illustrated by 151 problems in one publication which cover seven fields in elementary statistics without touching statistical quality control.[3] Brevity, however, in illustrations used for teaching statistical quality control is often paramount, and for this reason a series of short

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[3]Robert E. Chaddock and Fred E. Croxton, Exercises in Statistical Methods (Boston: Houghton Mifflin Company, 1928).

demonstrations have been carefully selected[4] to help in presenting some essential aspects of the subject.

A selection of the most appropriate demonstration to recommend for illustrating a given principle has been made on a weighted factor basis. The factors used were selected with the advice of the thesis advisor, and their relative weights have been assigned by the writer, based on his opinion of a compromise between those which would be considered important under industrial and academic conditions. The wide variety of circumstances, however, under which this type of instruction may be given makes even a compromise weighting of factors subject to question. The author is well aware that both factors and weighting are controversial, and submits the following basis of comparison for lack of better standards:

Factor	Symbol	Maximum Weight
Demonstration of Principle	P	12
Required Demonstration Time	T	10
Adaptability to Individual Participation	I	8
Similarity to Shop Practice	S	6
Economy of Cost of Equipment or Materials	C	4
Total of Five Factors		40

[4]These demonstrations have been chosen largely from suggestions submitted by many of the outstanding teachers of statistical quality control in industry and leading colleges. See Appendix, Tables IX and X.

It was considered that the foremost factor affecting the selection of any device should be its relative effectiveness to demonstrate the principle being emphasized. This was given the greatest weight--12 points.

The time required to complete the demonstration was considered to be of next importance, particularly in the mind of many industrial leaders. For no defensible reason the weight for this factor was set at 10 points, this representing in the mind of the writer a reasonable difference below, but comparable to, the first factor.

The third factor chosen was the relative adaptability of the demonstration to individual participation, prompted by the writer's belief that a student learns by doing. Only 8 points were assigned.

The similarity of the demonstration to shop practice or the practical tie-in with what can be expected in actual practice was selected as the fourth factor and weighted 6 points.

The last but by no means unimportant factor was cost or investment in equipment and materials. In some cases departments teaching elementary statistical quality control techniques may operate with limited funds, while in most industrial installations this factor is probably of less importance than in colleges. Because of the emphasis of this study on the needs and methods of industrial training, this factor is set at only 4 points.

In assigning these point values to various methods and devices for presenting the principles to be emphasized, the writer considered each factor only in three degrees of value:

Clearly advantageous	-	full weight
Advantages not outstanding	-	half weight
Clearly disadvantageous	-	no weight

The above weighting of factors has resulted in significant differences in some cases, but in others the point values are close enough to provoke question concerning the final choice of demonstrations. In these cases, no defense is offered to uphold the evaluation scheme. Alternate demonstrations are described, but only one recommendation is included.

CHAPTER II

CHARACTERISTICS OF A NORMAL DISTRIBUTION

Objective. The object of this demonstration is to illustrate some characteristics of a normal or constant cause system of distribution (variations which can be attributed to random chance).

Discussion. One of the first concepts to be mastered by one approaching statistical quality control for the first time is that man has never been able to intentionally reproduce any two articles which are exactly alike.[1] Some reproductions are so crude that the differences can be readily perceived by observation, but others are so nearly alike that very delicate adjustments of the most accurate measuring devices are required to distinguish their differences. In every realm of measurement, however, these variations, which result despite reasonable efforts to produce a given standard, tend to cluster about one value as a center. This type of distribution is said to result from chance variation in that all adjustable conditions were kept constant.

The point about which chance variations tend to centralize is known as the average, and this is the most common method of referring to values which are known to vary. Persons completely

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[1] Clifford W. Kennedy, Quality Control Methods (New York: Prentice-Hall, 1948), p. 3.

unfamiliar with statistical procedures seldom hesitate to speak of the "average" as a measure of temperature, of weight, or of such indeterminate characteristics as those of the "average man." This is properly called the mode,[2] the term given in statistics to that value which occurs most often. On the other hand, when other averages, such as gasoline mileage or baseball records, are spoken of, the same word is used for a more specific value, which in statistics is called the mean or arithmetic mean. The mean, hereafter denoted as $\overline{X}$ (pronounced "X-bar"),[3] is computed by dividing the sum of the values (X_1 X_2 etc.) by the number of items.[4]

Values of the variables can be expected to cluster or be distributed about $\overline{X}$ according to a definite pattern determined in each case by the factors affecting the variable. In the case of chance variation, the pattern tends to approach that of mathematical probability, which is known as normal distribution. This pattern is symmetrical about $\overline{X}$, with a larger number of values close above and below, and fewer values which "missed the average" by any considerable amount, as illustrated in Figure 1. One measure of the spread of the pattern above and below the mean has been established by what is called the

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[2]John F. Kenney, Mathematics of Statistics (New York: D. Van Nostrand Company, Inc., 1947), p. 47.

[3]See Page viii for symbols used in this work.

[4]Kenney, *op. cit.*, p. 33.

standard deviation. This is expressed in the same units of measurement as for the data,[5] and is denoted by the symbol σ (pronounced: sigma). By the use of σ and $\overline{X}$, it is possible to estimate what proportion of the measurements for a normal distribution will probably fall within any specified limits. This is a most useful relationship, the most frequently used values of which are that about two-thirds of the occurrences of X will fall within one sigma on either side of $\overline{X}$, all except about five percent within $\pm 2\sigma$, and practically all (99.73%) within $\pm 3\sigma$,[6] as illustrated in Figure 2.

When the variable X can have values which are not integers, e.g. X = 2.13854, there is an infinite number of possible values to which X may vary. It is possible to compute the mean and the standard deviation from this sort of data, even if no two values of X are exactly alike.[7] To construct a histogram for visualizing the distribution in this case, it is necessary to classify the data into groups called cells. There is often a problem as to how many cells there should be and what values should be included. A rough working rule used by

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[5] Paul G. Hoel, Introduction to Mathematical Statistics (New York: John Wiley & Sons, 1947), p. 11.

[6] Eugene L. Grant, Statistical Quality Control (New York: McGraw-Hill Company, 1946), p. 69.

[7] Using actual values, the sum divided by the number of specimens will yield $\overline{X}'$ for the population, and the sum of the squares of each X, $\sum X^2$, can be used to compute the standard deviation:

$$\sigma = \sqrt{\frac{\sum X^2}{n} - (\overline{X}')^2}$$

.8761
.8759
.8757
.8755
.8753
.8751
.8749

Figure 1

Frequency Distribution for Measurements of a Group of Machined Parts

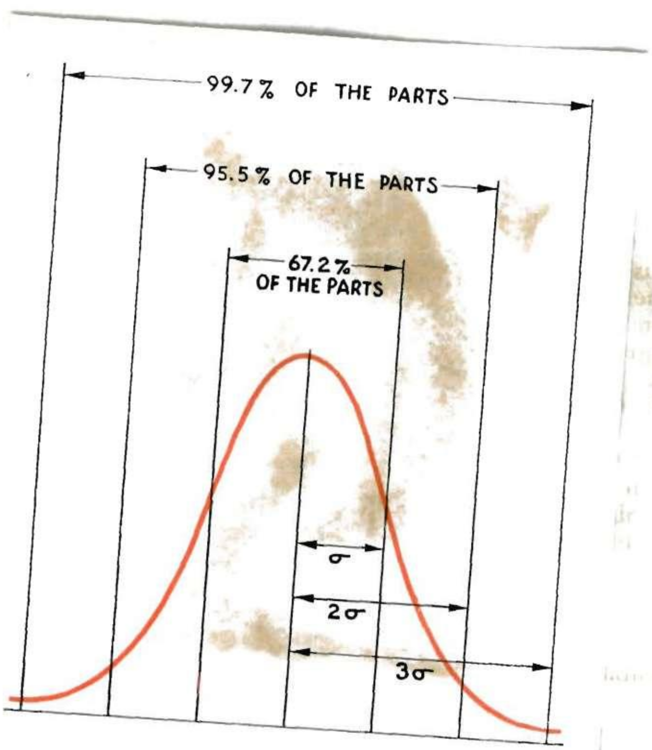

Figure 2

Percentages of Total Work Included Within Various Sigma Limits

Source: Timken Axle News, SP 4908, Published by Timken-Detroit Axle Company, Detroit, Michigan, page 3.

statisticians is to aim to have about 20 cells.[8] Each cell includes values between the cell limits of the accuracy of the measurements considered, and the cell intervals must be equal.

Analysis of existing techniques. The most common method of illustrating the concept of chance variation is the traditional one of using colored beads or marked chips drawn from a bowl or some mechanical mixing device.[9] Next in popularity is the use of various models of Galton's quincunx,[10] shown in Figure 3, which is being introduced into training programs in both colleges and industry. Other training aids frequently used include coins, dice, peg boards with washers, roulette wheels, slot machines, charts, slides or movies. One unusual aid for those who dislike plotting charts is a punchboard specially prepared for frequency counting. As the operator punches he unknowingly prepares a histogram consisting of holes.[11]

Considering each of the above in terms of the first criteria set forth in the introduction to this dissertation,

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[8]Grant, op. cit., p. 51.

[9]Based on the results of a survey of 44 colleges and 70 industrial organizations using statistical quality control. See Appendix, Tables IX and X.

[10]W. J. Jennett. Discussion of a paper by A. W. Swan, "Work and Organization of a Statistical Department in Heavy Industry," Iron and Steel Institute Journal, 162:172-5, June 1949.

[11]Arthur Bender, Jr., Delco Remy Division, G.M.C., personal letter, 8/8/50.

it is the opinion of the writer that those devices which depend upon the counting of probable differences, i.e. colored beads, coins, dice, roulette wheels, or slot machines, do not demonstrate the principle of chance variation as clearly as those which indicate a deviation about a specific central point.

The time required to accomplish a comprehensive demonstration is considerable where individual items have to be measured, e.g. peg boards which require measurement of each washer before assembly. Less time is required if the measurement is already written, as in the case of marked chips in a bowl; but when a cycle of motions is required to obtain each reading, only a limited number of items can be finished in a given length of time. The use of a mechanical chip mixer (see Figure 4) to speed up random drawings will permit not only a faster demonstration but a more random set of values. The quincunx, illustrated in Figure 3, is the fastest among the visual aids described in this study, in that a large number of balls can be dropped, classified, and totalled in a minimum of time. Another feature of the quincunx, and one which is unique, is the direct development of a histogram.

All of the devices listed above, except charts, slides, and movies, can be arranged with equal ease for individual participation.

It is the opinion of the writer that during the early stage of instruction strict conformance to shop practice is not

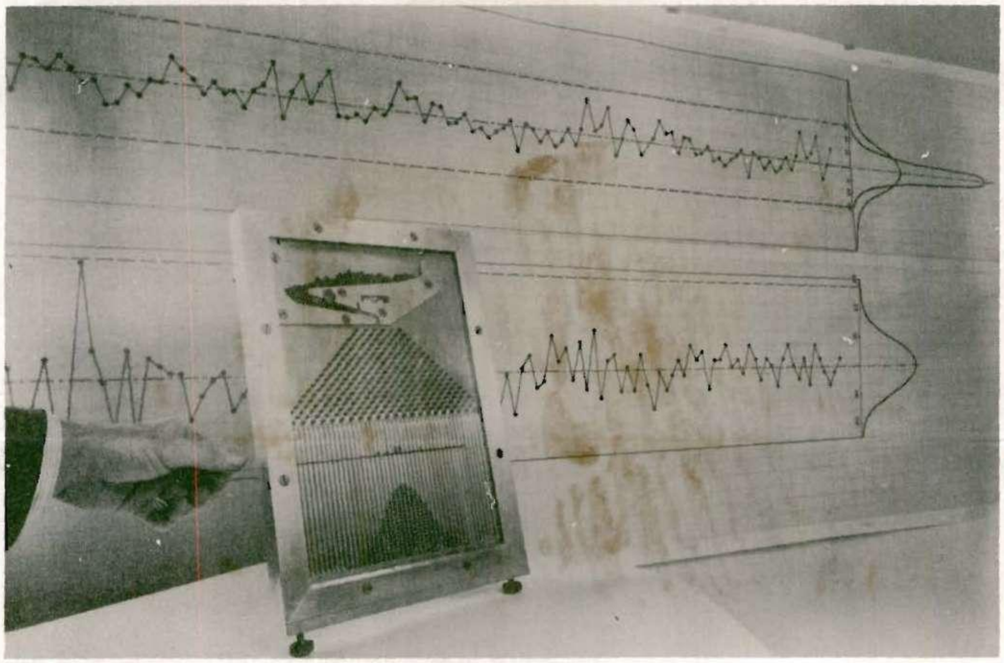

Figure 3

The Quincunx

The quincunx in the above photograph is manufactured by the Timken Roller Bearing Company of Canton 6, Ohio. It is approximately 10" x 14-1/2" x 1" overall and employs 500 one-eighth inch diameter balls. This small construction lends itself to effective 15 minute demonstrations in foreman's offices and to small groups of people.

A larger and more flexible version of this type equipment has been designed by Mr. Dorian Shainin at Hamilton Standard Propellers Division of United Aircraft Corp., and machines using his designs have been made by Mr. W. H. Scadden, 13 Courtland Street, Manchester, Connecticut.

essential, and none of the suggested devices can claim this advantage. Some industrial quality control engineers insist on keeping all instruction in the work bench atmosphere, by using articles taken from a production machine to illustrate normal distribution, disregarding the remote probability of such unrehearsed sampling to be normal.[12] If it is considered necessary to tie in the normal distribution to something practical, it is possible to cite the problem of how many shoes or men's suits should be stocked of each size to approximate the anticipated demand for that size.[13]

The cost of materials and equipment is relatively small for such items as beads, chips, coins, dice, washers, and charts. It may be possible to obtain roulette wheels or slot machines seized by police, and these can be adjusted to remove bias at little expense. Slides and movies represent only a moderate expenditure. The quincunx can be made locally or purchased complete. The cost can be spread over the demonstration expense for several experiments, however, for the equipment is readily used to illustrate different principles of quality control.

Despite the cost, it is recommended on a basis of weighted factors, shown in Table I, that the quincunx is

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[12] Kennedy, op. cit., p. 144.

[13] R. L. Hermann, The Ladish Company, Cudahy, Wisconsin, personal letter, 7/21/50.

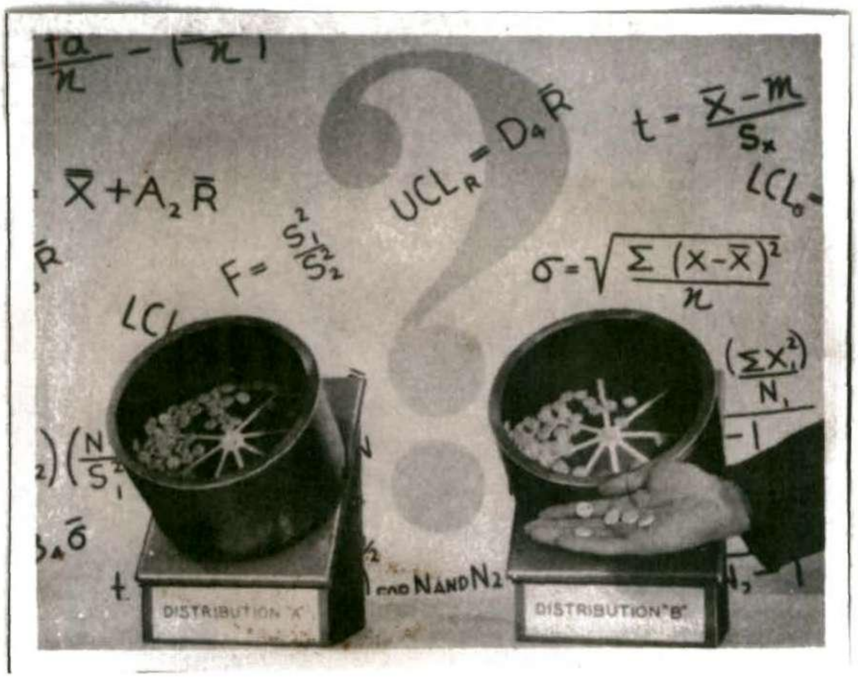

Figure 4

Mechanical Chip Mixers

The use of chip mixers adds drama to the demonstration of any statistical principles which are explained by the use of numbered chips. The continuous mixing of chips attracts and holds the attention of large audiences and speeds up any chip demonstration by mixing the chips rapidly and thoroughly. Tests show that random samples are obtained when samples are taken, read, returned to the mixer and another sample taken immediately.

TABLE I

APPRAISAL OF DEVICES FOR DEMONSTRATING NORMAL DISTRIBUTION

Demonstration	P	T	I	S	C	Total
Colored Beads	6	5	8	0	4	23
Marked Chips	6	10	8	0	4	28
Quincunx	12	10	8	0	0	30
Coins	6	5	8	0	4	23
Dice	6	5	8	0	4	23
Pegboards with Washers	12	0	8	0	4	24
Roulette Wheels	6	5	8	0	2	21
Slot Machines	6	5	8	0	2	21
Charts	12	10	0	3	4	29
Slides	12	10	0	3	2	29
Movies	12	10	0	3	0	29

Factors	Symbol	Maximum Weight
Demonstration of Principle	P	12
Required Demonstration Time	T	10
Adaptability to Individual Participation	I	8
Similarity to Shop Practice	S	6
Economy of Cost of Equipment or Materials	C	4
Total of Five Factors		40

probably the best device for illustrating the principle of chance variation and its relation to the normal distribution.

Apparatus required. Quincunx complete with set of balls, blank forms with appropriate number of columns for recording frequency for each column (see Figure 5).

Procedure. A histogram of the distribution of balls falling down through the peg maze of the quincunx can be constructed by releasing the entire supply of balls through a restricted opening at a central point, the classification bins at the bottom accumulating their respective quantities of balls diverted from the center. The height of each column or the number of balls in each bin should be recorded (whichever is most convenient) to form a basis of comparison with the normal distribution. The balls should be returned to the hopper at the top and released a second time, and the values again recorded, after which the performance can be repeated until a good average is established for each column. The averages should reveal a fairly smooth bell-shaped histogram which approximates the normal distribution, being symmetrical about the mean and tapering to very small values at the extremes. The quincunx can be marked for the location of plus and minus three sigma limits based on averages of several runs using identical conditions and the same number of balls. For about 800-1000 balls or less, practically all should fall within these limits.

Plus Values - Receptacles in Quincunx - Minus Values

Trials	10	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	10
1																					
2																					
3																					
4																					
5																					
6																					
7																					
8																					
9																					
10																					
11																					
12																					
13																					
14																					
15																					
Total																					
Averages																					

Figure 5

Form for Recording Frequency in Quincunx Observations

CHAPTER III

CHARACTERISTICS OF AVERAGES OF SAMPLES

Objective. The object of this demonstration is to point out certain characteristics of sample averages, and the relationships of their distributions to the original or parent population.

Discussion. One random sample of (n) values of a variable (X) will ordinarily include different values of X, and the mean of these values ($\overline{X}$), while set by the X's, will necessarily be closer to the mean for the whole population (denoted as $\overline{X}'$) than some of the values of X in the sample. See Figure . Because of this centralizing feature, $\overline{X}$ serves as a convenient measure of the sample. The larger the sample size, the closer will $\overline{X}$ of the sample approach $\overline{X}'$.[1]

The sample can also be measured by its range (R) which is the difference between the largest and smallest specimens in the sample. The range varies with the sample size, being larger as a general rule for larger samples. The range for any one sample is of little significance; however the mean range of several samples ($\overline{R}$) is a most useful measure of the spread of the total population. Mathematicians have prepared tables of conversion factors for computing the standard

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[1]Eugene L. Grant, Statistical Quality Control (New York: McGraw-Hill Book Company, 1946), p. 96.

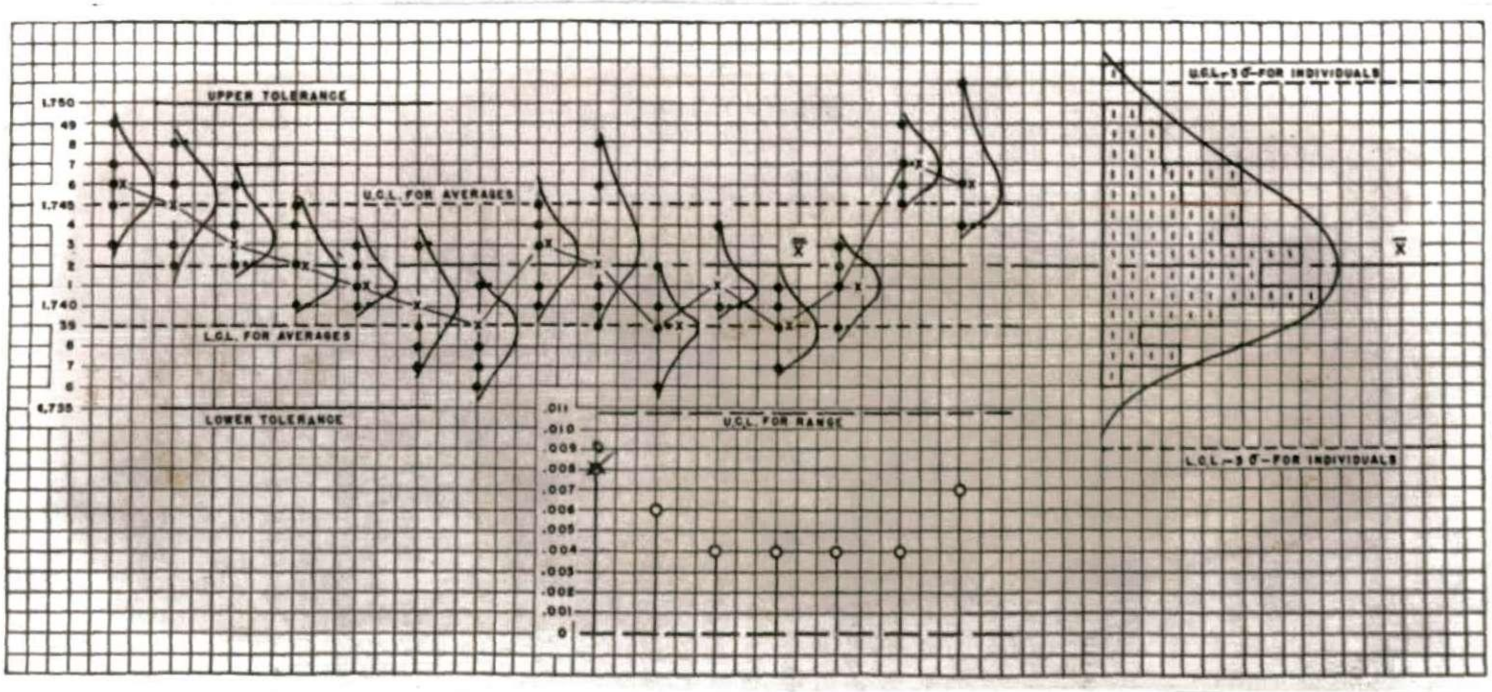

Figure 6

Combination of Average, Range, and Frequency Charts

Source: C. W. Kennedy, Quality Control Methods (New York: Prentice-Hall, Inc., 1948), p. 188.

deviation (σ) of a population in terms of the average range of a number of samples, based on the sample size (n specimens). To compute σ it is necessary merely to look up a factor[2] denoted as (d_2) for the appropriate value of n, and divide it into $\bar{R}$. This method of computing σ, while requiring less arithmetic, is no more accurate than that which requires several squares and a square root.

Mathematically it can be proved that σ of the $\bar{X}$ distribution (denoted by $\sigma_{\bar{X}}$) is not only less than the σ of the original population, but that it varies with the value of n thus: $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$. This standard deviation of the frequency distribution of $\bar{X}$ is often referred to as the standard error of the mean.

If random samples are taken from a population which is not normal, the $\bar{X}$ values tend to form a distribution of their own, which also approximates a normal distribution.[3] This characteristic is of particular importance in that it enables some useful features of a normal distribution to be applied to populations which are not normal.

<u>Analysis of existing techniques</u>. On the basis of the writer's investigations, the use of "Shewhart's bowls" has been found to be most widely considered as an almost

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[2]See page viii for Symbols, and Table III for formulas for Control Limits.

[3]Walter A. Shewhart, <u>Economic Control of Quality of Manufactured Products</u> (New York: D. Van Nostrand Company, Inc., 1931), p. 180.

traditional method of illustrating the tendency toward normal exhibited by $\overline{X}$ distributions of samples from square, triangular, and normal populations.[4] Playing cards serve as an alternative for testing a rectangular population, and selected articles from manufacturing processes are also used as normal for this demonstration. The relative effectiveness of using specimens versus $\overline{X}$ of samples is sometimes illustrated by a chart, by special templates, or a model.

Repeating a portion of the demonstration originally published by Walter A. Shewhart, or using the results of his 4000 drawings from bowls of triangular, rectangular, and normal distributions,[5] is not only clear but flexible. The numbered chips in the bowls can be mixed mechanically, as previously described,[6] to improve the randomness and at the same time speed up the demonstration. The use of cards or manufactured articles requires more time than the use of numbered chips because of the shuffling required in one case, and the measurements to be made in the other.

Individual participation in the demonstrations can be easily arranged for any of the above described methods.

If it is desired to emphasize similarity to shop conditions, one can use a carefully prepared set of manufactured

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[4]Based on results of survey. See Appendix, Tables IX and X.

[5]Shewhart, op. cit., Appendix II.

[6]Mechanical Chip Mixer illustrated in Figure 4.

articles for the normal population, inspection rejects (roughly bimodal triangular) for an abnormal distribution, and a prepared rectangular distribution consisting of four or more specimens of each dimension class or cell. While this illustration exhibits distinct similarity to shop conditions and clearly demonstrates the principles to be set forth, the time required to acquire substantial results is considerably greater than that required to use the numbered chips.

All materials considered thus far, i.e. cards, chips, and specimen manufactured articles, are readily available and inexpensive, and the only item of moderate expenditure is the mechanical chip mixer, which is optional and might not be suitable for mixing manufactured articles.

For demonstrating the relationships of $\bar{X}$ and the individual specimens, and as a basis for proving the superiority of the sample technique, the principles can be illustrated clearly by using either the chart, templates, or special model. For example, Figure 6 has the advantage of showing all that is needed on one chart, which allows a convenient comparison of all phases without disturbing the display. Templates,[7] while distinctive and perhaps easier to see than some charts, have the inherent quality of being "previously prepared" and in that sense cannot be used if individual participation is desired.

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[7] Arthur Bender, Jr., Delco-Remy Division, G.M.C., Anderson, Indiana, personal letter, 8/8/1950.

A unique device for showing relative sensitiveness of individual specimens versus sample averages has been invented by Professor John A. Henry,[8] which uses an etched plexiglass sheet to slide over a fixed background. This device has been perfected for table discussion, and while it may be superior as a visual aid, it has the same inherent disadvantage of being data from previous selections, and in addition is moderately expensive.

Considering the relative merits of each of these devices, as shown in Table II, it is the opinion of the writer that the use of selected items from a manufacturing process should be used if convenient. Fiber chips, numbered to represent different types of population distribution can be used with a mechanical chip mixer to achieve satisfactory results in cases where parts cannot be used from the plant.

Apparatus. Fiber chips made up in populations of at least 200 each, representing triangular, rectangular, and normal distributions; a deck of ordinary playing cards (if desired); and forms for recording observations; computing $\overline{X}$, R, and σ. Mechanical chip mixers (Figure 4) are recommended with the use of fiber chips, but are not essential.

Procedure. The comparison of results from analyses of three types of population distribution will require three phases of procedure.

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[8] John A. Henry, Professor, Mechanical Engineering Department, University of Illinois, Urbana, Illinois, personal letter, 7/27/1950.

TABLE II

APPRAISAL OF DEVICES FOR DEMONSTRATING CHARACTERISTICS OF AVERAGES OF SAMPLES

	P	T	I	S	C	Total
Distribution of $\bar{x}$ for different populations						
Drawing marked chips	12	5	8	0	4	27
Drawing from mixer	12	10	8	0	0	30
Cards	12	3	8	0	4	27
Measuring manufactured articles	12	0	8	6	4	30
Use of Shewhart's tables	6	10	0	0	4	20
Relationship of $\bar{x}$ to individual specimens						
Kennedy chart	12	10	0	0	2	24
Templates	12	10	0	0	2	24
Model of special case	12	10	0	0	0	22

Factors	Symbol	Maximum Weight
Demonstration of Principle	P	12
Required Demonstration Time	T	10
Adaptability to Individual Participation	I	8
Similarity to Shop Practice	S	6
Economy of Cost of Equipment or Materials	C	4
Total of Five Factors		40

The first phase uses a mechanical chip mixer, and a counted normal distribution of 200 or more fiber disks. When the disks are well mixed, a sample of five disks is taken by hand and the values recorded on the appropriate form. The sample is then returned to the bowl and the entire population mixed thoroughly before withdrawing a second sample of five. The $\overline{X}$ and R are calculated for each sample (subgroup) of five, and sampling is continued until representative values for $\overline{\overline{X}}$ and $\overline{R}$ can be computed (at least 50 samples). Values of $\overline{X}$ are then classified and a frequency table and histogram drawn to show how close the values of $\overline{X}$ approximate a normal distribution.

The second phase is identical to phase one except that a triangular population of disks is used instead of a set having normal distribution.

The third phase uses a deck of playing cards dealt five cards at a time from the shuffled deck, which represents a rectangular population. Values are recorded using ace--1, jack--11, queen--12, and king--13. Cards are then inserted in the deck individually, and the deck reshuffled before taking a second sample of five cards. As described in phases one and two, $\overline{X}$ and R for each sample are computed and a histogram of $\overline{X}$ values plotted to compare with the results of the other two phases.

CHAPTER IV

CONSTRUCTION AND CHARACTERISTICS OF $\overline{X}$ AND R CHARTS

Objective. The objective of this project is the determination of control limits for $\overline{X}$ and R Charts, and the study of how the control charts work.

Discussion. After a production process has completed a given lot of items, the acceptability of the lot can be determined by checking each piece against a standard or, as was discussed in previous projects, by sampling. The conclusions in either case might be termed post-mortem, in that the defectives are unacceptable and it is too late to correct the cause of deviation. If samples are taken periodically during the process and plotted on a control chart, it is possible to predict troubles before they become serious. When abnormal variations are spotted in time, adjustments can be made which will keep the number of rejects to a minimum on the subsequent items. The $\overline{X}$ control chart technique grows out of the relationship of $\overline{X}$ and R that is used to predict how many items can be expected to fall within plus or minus three sigma limits, described in Chapter II. These probable limits for chance variation should be less than the spread between the required tolerances for that measurement.

Control limits are used somewhat similarly to the way an oil pressure gage is used by an automobile driver. As long as the indicator averages about 60, with only slight variations above or below the average, no attention is required (see Figure 7). If, however, the gage reading slips outside the driver's mental control limits, he will not wait for the ultimate consequences of driving without oil but will stop the car immediately to investigate the cause of the trouble.[1] The control chart likewise serves as a warning whenever a point approaches the control limits. If the point exceeds the limit, it is said to be "out of control."

The $\overline{X}$ chart shows the limits of $\overline{X}$ variation above and below the mean of the sample averages ($\overline{\overline{X}}$). These limits use the A_2 factor[2] multiplied by $\overline{R}$, or UCL = $\overline{\overline{X}} + A_2\overline{R}$, and LCL = $\overline{\overline{X}} - A_2\overline{R}$. Limits for $\overline{X}$, R, and p charts can also be readily computed by using the Bender Control Limit Calculator,[3] which is a disk type similar to a circular slide rule. These limits correspond to $\pm 3\sigma$, which includes about 99.7% of all values. This means that the probability of 0.997 or 997 chances out of a thousand of any point falling outside of the control limits will have been the result of either a shift of the distribution from its central point or a change in the spread of

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[1] Management Development Program, Statistical Quality Control Leader's Guide, Course 1067 (Dearborn, Michigan: Training Department, The Ford Motor Company), Session I, p. 25.

[2] See Table III for formulas for Computing Control Limits.

[3] Arthur J. Bender, "The Bender Control Limit Calculator," Industrial Quality Control, May 1950, p. 81.

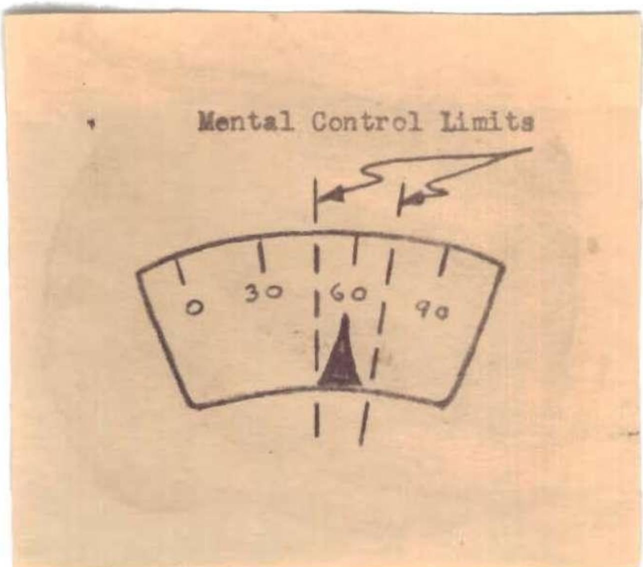

Figure 7

Control Limits for Automobile Oil Gage

Source: Management Development Program, Statistical Quality Control Leader's Guide, Course 1067 (Dearborn, Michigan: Training Department, The Ford Motor Company), Session I.

the distribution. The relative spread of $\overline{X}$ compared to X is found in $\sigma'_{\overline{X}} = \frac{\sigma'}{\sqrt{n}}$ mentioned in Chapter III. Therefore, evidence of an $\overline{X}$ beyond $\pm 3\sigma'_{\overline{X}}$ indicates a shift of $\overline{\overline{X}}$ more quickly and just as positively as the shift can be detected by the use of many single specimens. Any shift of the distribution as a whole will result in a change in $\overline{\overline{X}}$, and the control limits help spot such a shift. For example, if, on Figure 8, a point C occurs outside of the lower control limit for distribution A, it is probable that the distribution pattern A has shifted to another position such as B, for which C comes within limits. It is necessary to find what condition has caused the shift of distribution and correct it to bring succeeding points into control.

Another cause of loss of control is a change in the process which upsets the spread of the distribution pattern. This might be illustrated by Figure 9, in which the position of the mean does not change, but the distribution pattern flattens out, spreading the control limits to include values formerly beyond $\pm 3\sigma'$. For example, point D is beyond the lower control limit for distribution A, but it is within the limits for distribution B. This kind of change in pattern may not affect $\overline{\overline{X}}$, but both σ and R will be increased. By plotting the R (or the σ) for each sample, it is possible to spot this kind of loss of control quickly.

TABLE III

FORMULAS FOR COMPUTING CONTROL CHARTS

Control with No Standard Given
(Subgroup Data Used to Provide Standard)

	Average $\bar{X}$	Standard Deviation σ	Range R	Fraction Defective p
Upper Limits	$\bar{\bar{X}} + A_1\bar{\sigma}$ $\bar{\bar{X}} + A_2\bar{R}$	$B_4\bar{\sigma}$	$D_4\bar{R}$	$\bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$
Central Lines	$\bar{\bar{X}}$	$\bar{\sigma}$	$\bar{R}$	$\bar{p}$
Lower Limits	$\bar{\bar{X}} - A_1\bar{\sigma}$ $\bar{\bar{X}} - A_2\bar{R}$	$B_3\bar{\sigma}$	$D_3\bar{R}$	$\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

Control with Standard Given
(Standards Provided)

	Average $\bar{X}'$	Standard Deviation σ'	Range ($\bar{R}' = d_2\sigma'$)	Fraction Defective p'
Upper Limits	$\bar{X}' + A\sigma'$	$B_2\sigma'$	$D_2\sigma'$	$p' + 3\sqrt{\frac{p'(1-p')}{n}}$
Central Lines	$\bar{X}'$	$c_2\sigma'$	$d_2\sigma'$	p'
Lower Limits	$\bar{X}' - A\sigma'$	$B_1\sigma'$	$D_1\sigma'$	$p' - 3\sqrt{\frac{p'(1-p')}{n}}$

Source: John W. Dudley, Examination of Industrial Measurements (New York: McGraw-Hill Book Company, Inc., 1946), p. 45.

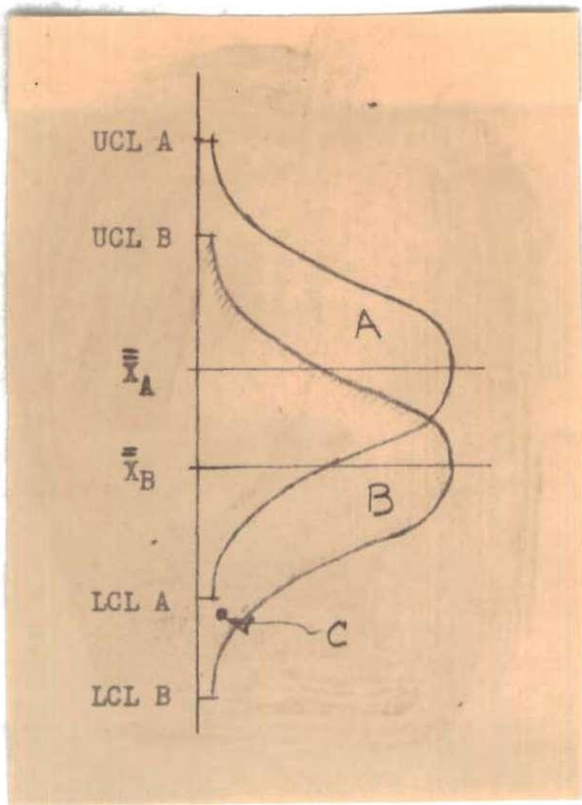

Figure 8

Effect of Shift of Population

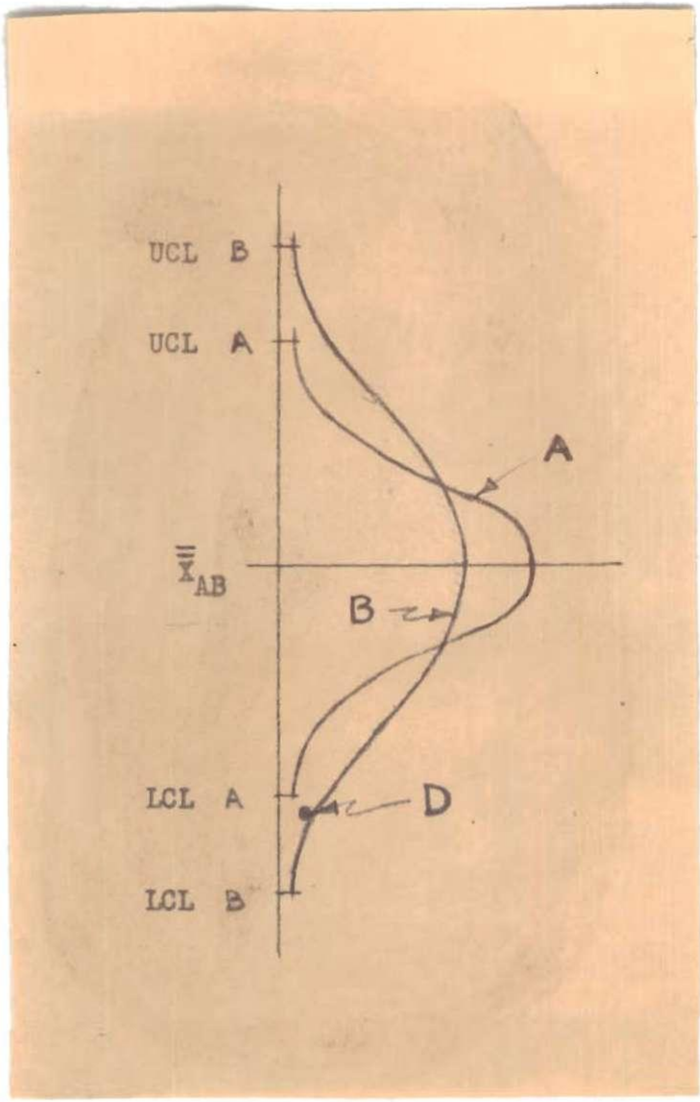

Figure 9

Effect of Change in Characteristic of Population

The limits for the R chart are computed, using a D_4 factor[4] for the UCL and a D_3 factor for the LCL. The $\overline{R}$ serves as the central line for the chart, and is multiplied by the D_3 and D_4 factors to obtain the limits.

As long as periodic samples yield values of $\overline{X}$ and R which fall within the control limits on the charts, it is reasonable to expect satisfactory production, and no additional inspection should be necessary for that particular measurement which is in control. If plotted values of $\overline{X}$ and/or R tend to approach either control limit, they can serve as warning to investigate the operation before it gets out of control. Adjustments made as a result of such warnings often make it possible to keep an operation in continuous control.

Analysis of existing techniques. The $\overline{X}$ and R charts are illustrated by using many types of demonstration equipment, including: numbered chips drawn from a bowl or chip mixer, articles taken from a manufacturing operation, wooden beads strung on wires, dice, the quincunx, or just a practice control chart at the operator's bench.[5]

Probably the most widely used method of demonstration is the drawing of numbered chips from a bowl. Values

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[4]See Table III for this and other formulas for computing control charts.

[5]See Appendix, Tables IX and X.

representing measurements are marked on fiber chips which are well mixed in a bowl, and samples are drawn to represent measurements taken periodically during production. The $\bar{X}$ and R for several samples form the basis for setting control limits and plotting the chart. If additional chips with extreme values can be mixed into the bowl gradually, the control chart will reveal the change within a few samples. This method of presentation is economical, effective, and can be carried on relatively fast by using a mechanical chip mixer.

The next most popular method of demonstrating $\bar{X}$ and R charts, from the writer's investigation, is apparently the use of actual manufactured articles taken from a production operation. This is far more practical than the use of chips because the readings are directly comparable with daily experiences in the plant. Defective pieces can be mixed into the set to show the effect on the control charts.[6]

Dice, especially when used with a phony set, are used effectively in many plants. The phony set has two each of 4, 5, and 6, instead of the normal numbering, and is used to throw the chart out of control.[7] The principles of control

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[6]This method is enthusiastically endorsed by some authorities, including J. G. Vanhoy, Detroit Diesel Engine Div., General Motors Corp., and C. W. Kennedy, Federal Products Corp., Providence, R. I. Personal letters, 7/26, 7/28/50.

[7]Used by several large industrial organizations. See Appendix, Table X.

chart construction and operation can be demonstrated with dice, but there is no time saved over the use of chips, and this procedure is difficult to make compatible with everyday industrial practice.

The relationship of samples to the normal distribution pattern is effectively demonstrated by using the quincunx in many plants. A sample of balls is released through the peg maze into the cells of the quincunx in the normal manner, but is stopped partway down in order that the position of the balls be noted before they are released into the bottom of the cells. This procedure takes considerable time and slows down this device to a pace below that of some of the other methods.

A special piece of apparatus designed for this demonstration consists of beads strung on parallel wires simulating elements of a control chart. See Figure 10. The wires are strung on a frame which can be displayed horizontally to plot the chart and then turned 90 degrees to allow the beads to slide to one end, forming a histogram.

Articles taken from one point in a production process, or purchased items such as small roofing nails, can be measured for length. These two demonstrations are recommended on the basis of the evaluation shown in Table IV. It is recommended that measurements be plotted on a special bead and wire display, as illustrated in Figure 10. If nails, for example, are used, defectives to upset control can be prepared beforehand by filing a small amount from the points.

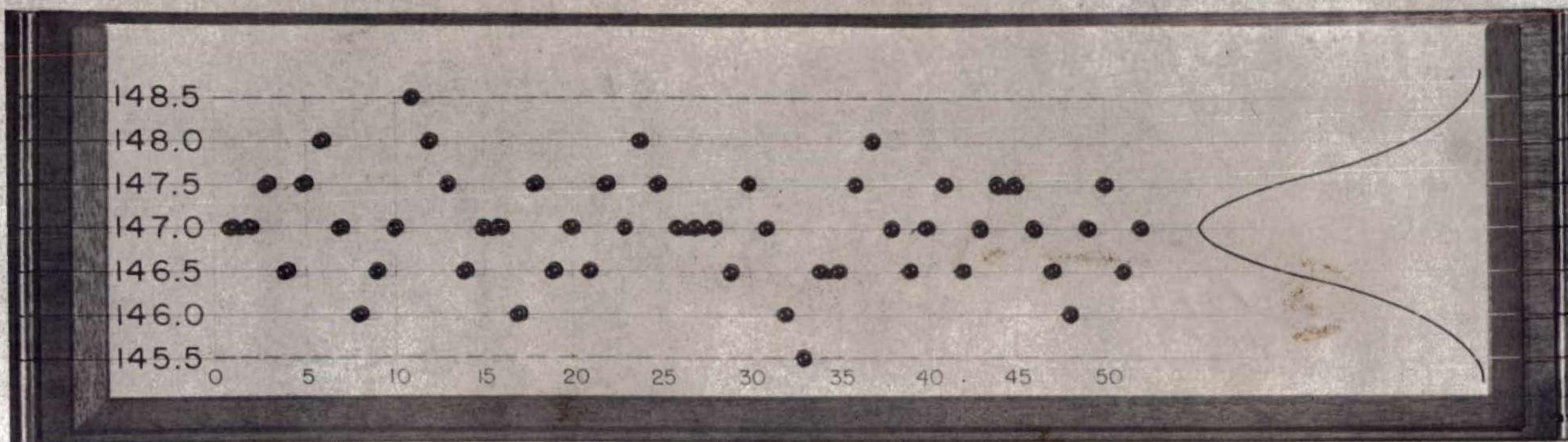
148.5
148.0
147.5
147.0
146.5
146.0
145.5
0
5
10
15
20
25
30
35
40
45
50

Figure 10

Control Chart Demonstrator

Photo by Anaconda Wire and Cable Co., Inc., Hastings-on-Hudson 6, New York

TABLE IV

APPRAISAL OF DEVICES FOR DEMONSTRATING

$\bar{X}$ AND R CHARTS

	P	T	I	S	C	Total
Demonstration						
Drawing chips from bowl	12	5	8	3	4	32
Drawing chips from mixer	12	10	8	3	0	33
Articles from manufacturing process	12	5	8	6	4	35
Dice	12	5	8	0	4	29
Quincunx	12	5	8	3	0	28
Separate aids in $\bar{X}$ and R chart instruction						
Wooden beads on wires	12	10	4	3	2	31
Chart at worker's bench	12	5	8	6	4	35

Factors	Symbol	Maximum Weight
Demonstration of Principle	P	12
Required Demonstration Time	T	10
Adaptability to Individual Participation	I	8
Similarity to Shop Practice	S	6
Economy of Cost of Equipment or Materials	C	4
Total of Five Factors		40

Apparatus. Small roofing nails; micrometer calipers; recording sheet; bead and wire control chart board.

Procedure. Samples of five nails are drawn from a bin and the length of each measured to the nearest thousandth with micrometer calipers. The values are recorded and checked, and the range is computed and plotted on a range chart by using a crossmark.[8] The average of five lengths is computed as $\overline{X}$ and the value plotted on an $\overline{X}$ chart, using a large round dot.[9] Successive samples of five are drawn and the values plotted for $\overline{X}$ and R for each, until at least 25 points have been plotted.

Trial limits for the R chart should be worked out first. The average range ($\overline{R}$) is computed by dividing the total of the ranges by the number of samples. This value of $\overline{R}$ is plotted as the central line for the R chart. The control limits for the R chart are found by using the D_3 and D_4 factors from a standard text.[10]

$$UCL_R = D_4\overline{R}$$
$$LCL_R = D_3\overline{R}$$

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[8] Paul Peach, An Introduction to Industrial Statistics and Quality Control (Raleigh, North Carolina: Edwards and Broughton Company, 2nd Edition, 1947), p. 98.

[9] Peach, loc. cit.

[10] For example: Eugene L. Grant, Statistical Quality Control (New York: McGraw-Hill Company, 1946), Appendix III, Table III; Clifford W. Kennedy, Quality Control Methods (New York: Prentice-Hall Company, Inc., 1948), p. 183; Edward S. Smith, Control Charts--An Introduction to Statistical Quality Control (New York: McGraw-Hill Book Company, Inc., 1947), Table III, p. 158.

The upper and lower trial control limit lines can be drawn in and extended beyond the first group of points, thus completing the form for the R chart.

The average of the $\overline{X}$ values (sum of $\overline{X}$'s, divided by the number of samples) is known as $\overline{\overline{X}}$ and becomes the central line for the $\overline{X}$ chart. The trial limits are found by use of the A_2 factor taken from a standard text on the subject,[11] or computed by use of the Bender calculator (see Chapter IV).

$$UCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R}$$

$$LCL_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R}$$

These trial limits can be drawn on the chart and the lines extended for use beyond the trial samples. Additional samples are then drawn and the values of $\overline{X}$ and R plotted. Short nails may be added to the bin so that the plotted points on the $\overline{X}$ chart will indicate the trend toward lack of control. The effect of long nails will be to throw the $\overline{X}$ chart out of control in the opposite direction, while the combination of long and short nails added to the bin will affect the R chart perhaps more than the $\overline{X}$ chart.

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[11] Ibid.

CHAPTER V

CONSTRUCTION AND CHARACTERISTICS OF p AND np CHARTS

Objective. The objective of this project is the determination of control limits for p and np charts and the demonstration of how these charts work.

Discussion. The $\overline{X}$ chart is a control procedure for measurements which fluctuate above and below some average value; however, some inspections are not based on such continuous variables, but instead measure only attributes. Examples of attributes are the inspection of chinaware which either is or is not cracked, castings which are complete or incomplete, or cloth which is determined to be either stained or clean.

To set up a control chart for this type of variation it is necessary to base the limits on previous samples, using probability as a basis for determining the limits of fluctuation. Ordinarily the relative number of specimens with unsatisfactory attributes is very small when compared with the number of satisfactory items, and thus a relatively large sample may be required before a specimen appears which contains the defective attribute. When the samples are not only large, but of unequal size, the ratio of the number of specimens with the particular attribute to the total number in the sample is designated as p (sometimes called the fraction defective) or

$p = \frac{\text{no. of defective}}{\text{total sample}}$. An everyday terminology of "percent defective" is therefore equal to 100 p. The standard deviation[1] $\sigma = \sqrt{npq}$ for $\sigma_p = \frac{\sqrt{npq}}{n} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{p\ (1 - p)}{n}}$ or for experimental data $\sigma_p = \sqrt{\frac{\bar{p}\ (1 - \bar{p})}{n}}$.

Limits for the p chart are calculated from $\bar{p}$ of a number of previous trials, through the application of the formula $\sigma_p = \sqrt{\frac{\bar{p}\ (1 - \bar{p})}{n}}$. The control limits are therefore $\bar{p} \pm 3\sqrt{\frac{\bar{p}\ (1 - \bar{p})}{n}}$. Often the chart is drawn using (100·p) to take advantage of the popular understanding of percentage defective. When n varies from day to day and limits are computed for each sample it is convenient to use the second form of the equation for computing limits: $3\sqrt{\frac{\bar{p}\ (1 - \bar{p})}{n}} = \frac{3\sqrt{\bar{p}\ (1 - \bar{p})}}{\sqrt{n}}$. The upper portion of the fraction can be calculated once, and values of $\sqrt{n}$ divided into it, to compute the limits as the sample size changes from day to day.[2]

In cases where the sample size can be controlled, it is easier to fix the sample size[3] and merely count the number of

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[1]Eugene L. Grant, Statistical Quality Control (New York: McGraw-Hill Book Company, 1946), p. 235.

[2]Limits can be easily obtained from special charts. See Holbrook Working, A Guide to Utilization of the Binomial and Poisson Distributions in Industrial Quality Control (Stanford University, California: Stanford University Press, 1944).

[3]"The sample should be large enough so that at least nine times out of ten one or more defectives will be found." William B. Rice, Control Charts in Factory Management (London: John Wiley and Sons, Inc., 1947), p. 81.

defectives per sample. This number of defectives is designated as np:

$$p = \frac{\text{no. defective}}{\text{total sample}}; \; p = \frac{\text{no. defective}}{n} \quad \text{no. defective} = np$$

No calculations are needed to compute values for an np chart inasmuch as the number of defectives can be plotted directly. Such a chart can be designated as a "chart of number defective--per sample of _____." The σ for np can be easily computed from $\sigma_{np} = \sqrt{npq} = \sqrt{n\overline{p}\,(1 - \overline{p})}$. The control limits are therefore $\overline{np} \pm 3\sqrt{n\overline{p}\,(1 - \overline{p})}$. The worker is probably better able to understand and utilize this kind of chart for attributes.

Analysis of existing techniques. Results of the investigation by the author disclosed only a few special devices to illustrate the p and np chart. Outstanding in this regard is the "Quality Control Indicator" developed and distributed by the General Electric Company. This device (shown in Figure 11) reduces the p chart to a simple continuous dial indicator reading which can be set up at a distance and yet read at a glance. The cost of this type of installation is considerable, but continuous centralized control is possible for several inspection points simultaneously.

The most popular teaching device for instruction in p and np charts, as indicated by the replies,[4] was the use of colored beads. Reference to this kind of control chart was also

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[4]See Appendix, Tables IX and X for results of survey.

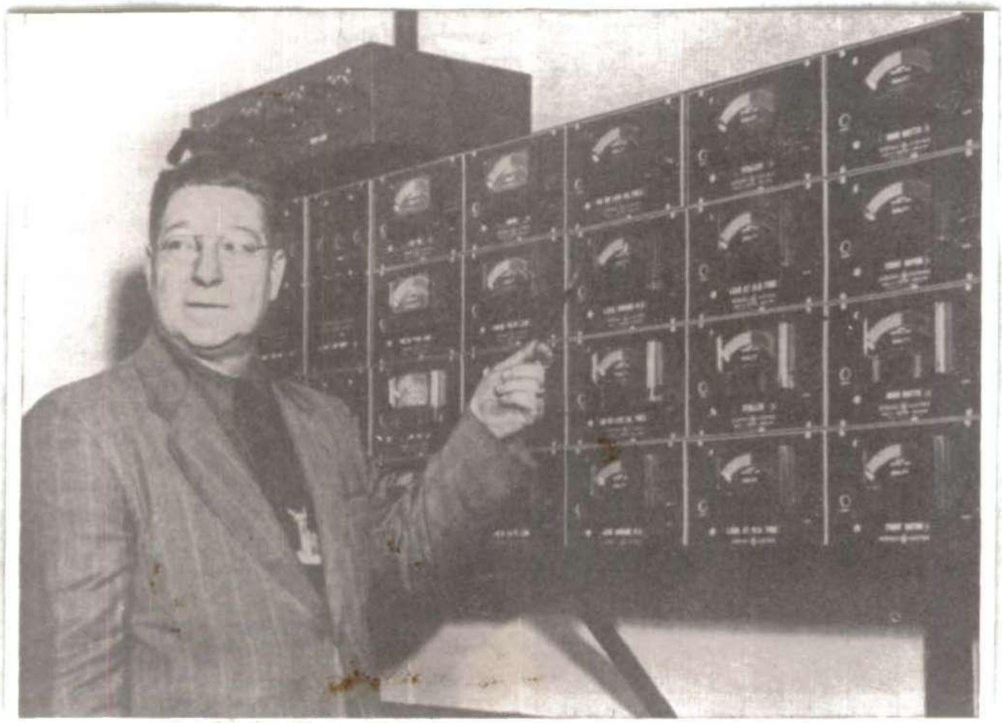

Figure 11

An Installation of Quality-Control Indicators for controlling Twenty Quality Characteristics on four production lines

Source: R. C. Miles, "Quality Control Indicator," Mechanical Engineering, Vol. 72, No. 3, March 1950, p. 224.

indicated by those who recommended marked chips. One contributor explained how the use of special marked chips of the material with which industrial trainees were working (e.g. wood chips in a woodworking plant) helped to spur interest above the use of ordinary fiber chips. Several industrial concerns indicated that they use sample parts taken from their own production to illustrate control of this type. The use of dice, cards, and prepared charts was indicated as being used in some instances.

Comparison of the same factors used for evaluating devices in previous demonstrations indicates the drawing of colored beads as the most appropriate or recommended procedure. (See Table V). It is noteworthy, however, that the use of marked chips made of appropriate materials and the use of selected manufactured parts compare favorably.

<u>Apparatus</u>. Standard known population of colored beads (980 white - 20 red or other appropriate combination); mixing box; and 50 hole sampling paddle; additional colored beads for adjusting the population to illustrate "out of control"; p and np charts (or material for making these charts).

<u>Procedure</u>. A handful of beads is taken from the box of known population and counted, and the number of colored defectives is noted. The p (or fraction defective) is computed by dividing the number defective by the sample size. Using p

TABLE V

APPRAISAL OF DEVICES FOR DEMONSTRATING

CONSTRUCTION AND CHARACTERISTICS OF p AND np CHARTS

Demonstration	P	T	I	S	C	Total
Drawing colored beads	12	10	8	3	4	37
Drawing card samples	12	5	8	0	4	29
Using marked chips	12	5	8	3	2	30
Chips of Appropriate Material	12	5	8	6	4	35
Dice	6	5	8	0	4	23
Manufactured parts	12	5	8	6	4	35
Charts	12	10	0	3	2	27

Factors	Symbol	Maximum Weight
Demonstration of Principle	P	12
Required Demonstration Time	T	10
Adaptability to Individual Participation	I	8
Similarity to Shop Practice	S	6
Economy of Cost of Equipment or Materials	C	4
Total of Five Factors		40

and the sample size n, trial control limits can be calculated. All three points are plotted on the chart, i.e. the UCL, LCL, and p for the sample.

The sample is returned to the box and the population thoroughly stirred; then a second handful of beads is withdrawn and the procedure repeated. The new sample control limits and the corresponding value of p are plotted, and the plotted points connected by a line.[5]

After several samples have been plotted (approximately 20 for this demonstration), the average subgroup size and the average p can be estimated, based on these samples, and trial limits computed for use in plotting additional samples.

Additional handfuls can be counted and the p for each plotted on the chart indicating control, if the sample sizes stay close to the average value of n.

To test control, 10 colored beads are added to the box, stirred well and 3 or 4 samples are taken as before; then 10 more colored beads are added, and the effect on the control chart noted.

The np chart is next plotted for a constant sample size (50 beads in sampling paddle), and the values for np (actual count of colored beads in each sample) are plotted directly and joined as in the case of the p chart. After several values of

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[5]Grant, loc. cit., p. 278.

np have been plotted, the $n\bar{p}$ or average number of defectives and the control limits can be computed. Tests for control can be made by removing colored beads from the population (or adding to if desired) to note the effect on the chart.

CHAPTER VI

CONSTRUCTION AND CHARACTERISTICS OF THE C CHART

Objective. The object of this demonstration is the determination of control limits for the c chart and the demonstration of an application of this type chart.

Discussion. The c chart is a variation of the np chart discussed in Chapter V. The "number of defectives" (pn) is often a convenient measure of quality when all samples comprise the same number of articles (i.e. when n is constant).[1] However, in setting up a control chart for "number of defects" in a sample, the c chart is used. "The sample may be a single article or specimen, a designated specimen length or area, a sample comprising a designated number of articles or specimens, etc."[2]

It is also understood that samples are always of equal size so that the expected number of defects is the same for each subgroup. The central line on the c chart is $\bar{c}$, which is the total number of defects divided by the number of samples; e.g. if on 30 test specimens a total of 18 defects were found (perhaps several defects on a single specimen), then $\bar{c}$ would

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[1] American War Standards. Control Chart Method of Controlling Quality during Production (Z1.3-1942) (New York: American Standards Association, 70 East 45th Street), p. 17.

[2] Ibid., p. 18.

be 18/30 = 0.60, which would be the central line for the chart.

The limits are computed from the general formula: $\sigma_{np} = \sqrt{n\bar{p}\,(1 - \bar{p})}$. For individual samples, np = c, so that $n\bar{p} = \bar{c}$, and substituting this in the formula yields $\sigma_c = \sqrt{\bar{c}\,(1 - \bar{p})}$. The ratio of defects to the total ($\bar{p}$) is assumed to be very small, e.g. less than 0.05, at which the expression $(1 - \bar{p})$ approaches a value of unity. This simplifies the expression to $\sigma_c = \sqrt{c}$. The control limits therefore are $\bar{c} \pm 3\sqrt{\bar{c}}$ in which c is never negative. These limits are easily computed and the c chart can be adapted to a wide variety of inspections, some of which are difficult to evaluate with more conventional measurements.

Analysis of existing techniques. Demonstration of the construction and use of the c chart has in some instances been confined to a narrative involving real or imaginary inspection values, their analysis, and their eventual combination into a c chart. This has been sometimes improved by the presence of actual samples of the inspected parts from which "on the spot" evaluations are used to compute $\bar{c}$ and construct a c chart. Special charts are also prepared to supplement this procedure, when actual measurements cannot be made at the demonstration.

A most effective device for mechanically applying the c chart inspection technique can be found in the "Quality Control Indicator" (see Figure 11) manufactured by the General Electric

Corporation. This machine is a combination of an electrical counting device and an accumulative computer which determines ratios for any number of inspections, e.g. the ratio defective, defectives per sample, or defects per sample, and indicates a continuous record on a meter which can be located in the office of the quality control inspector. It is an expensive device which can solve the problem in an otherwise difficult situation where immediate and continuing knowledge of quality is imperative. While this device is most effective, it is too expensive to be of value as a training medium.

As shown in Table VI, factors for evaluating the relative worth of demonstration devices to illustrate the c chart indicate that the use of manufactured parts and marked chips is most favorable. Inasmuch as the chips can be prepared in advance to eliminate counting, less time is required to conduct the experiment with them than with actual production items, but this gain is only at the expense of loss of shop atmosphere.

<u>Apparatus</u>. Prepared set of fiber disks with approximate Poisson distribution[3]; mechanical chip mixer; sheet for recording readings; and blank control chart sheet.

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[3]The $\sqrt{c}$ limits are the deviation of the Poisson. To set up a population of this type, the probability of x defects for a single disk $P_x = e^{-np}\frac{(np)^x}{x!}$ can be used by assuming a value for np. Assuming np = 0.1 yields P_o = 0.90484 which calls for 181 disks with zero defects in a population of 200. Similar computations yield 18 disks with 1 defect and 1 disk with 2 defects.

TABLE VI

APPRAISAL OF DEVICES FOR DEMONSTRATING CONSTRUCTION AND CHARACTERISTICS OF THE C CHART

Demonstration	P	T	I	S	C	Total
Charts	12	5	0	3	2	22
Manufactured Parts	12	5	8	6	4	35
Marked Chips	12	10	8	3	4	37
Quality Control Indicator	6	10	4	6	0	26

Factor	Symbol	Maximum Weight
Demonstration of Principle	P	12
Required Demonstration Time	T	10
Adaptability to Individual Participation	I	8
Similarity to Shop Practice	S	6
Economy of Cost of Equipment or Materials	C	4
Total of Five Factors		40

Procedure. A sample of 20 chips is drawn from the mixer; and the total of values on chips in the sample is noted. The total of values appearing on drawn chips is "total defects" in the sample, or c. The chips are replaced, mixed, and another sample drawn and checked for "defects," the total being recorded as c for the sample. This procedure is repeated until 20 or more samples have been checked, at which point the average value ($\bar{c}$) can be computed. This becomes the central line on a c chart, the upper and lower limits for which are $\bar{c} \pm 3\sqrt{\bar{c}}$. Any negative values for the lower control limit are considered as zero.

CHAPTER VII

THE VALIDITY OF 100% SAMPLING

Objective. The objective of this demonstration is to show that 100% inspection is not ordinarily 100% effective.

Discussion. To the uninitiated it seems logical that the best way to assure the elimination of defective parts is by thorough inspection of every specimen. This is known as 100% inspection. If the inspector must continuously inspect large numbers of items, the task becomes monotonous. To partially counteract boredom and the accompanying mental fatigue, a pattern of rhythm for necessary movements is often developed. Once a mechanical movement pattern is set, the inspector can increase his output, but only at the risk of other difficulties. For instance, finding it easy to continue automatically the rhythmic movement pattern, he may become less alert and allow defective parts to pass. Again, the inspector may develop in his mind a number pattern whereby he may automatically pass all specimens between certain numbers, e.g. since the last 2 defectives were the tenth items, every tenth item will be rejected as defective. This pattern may vary to include a predominating reading of odd or of even values.

Mental reaction to fatigue may cause the inspector to throw defective specimens into the accepted bin, after he has carefully separated one group from the other; on the other hand, the strain of continuous attention may cause him not only to permit defective items to pass but to discard acceptable items as being defective.

In a carefully controlled test,[1] one hundred defective items were mixed into a large acceptable lot. The first inspector spotted 68 of the 100 defectives; the second inspector spotted 18 of the remaining 32 defectives; the third inspector spotted 8 of the remaining 14 defectives; finally, a specially selected team of inspectors found 4 of the remaining 6 defectives. The results show that 400% inspection was not 100% effective and that 100% inspection allowed from 2-32% of the defectives to slip through.

In another test,[2] 61 lots of incoming material containing an average of 350 pieces were given 100% inspection and were also checked by measurements on samples of 50 drawn at random from each lot. The 100% inspection discovered only 4 defectives in 3 separate lots, which were not found by the sampling method; however, the sampling method rejected 34% of the lots which contained more than 500 defective pieces not discovered by the 100% inspection method. Better results with 30% lower total inspection cost were obtained by sampling.

Analysis of existing methods. The device constructed by Mr. Wagenhals of Timken Roller Bearing Company, illustrated

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[1] C. W. Kennedy, "Statistical Quality Control," Factory Management and Maintenance, 108:136-8, January 1950.

[2] P. L. Alger, "The Growing Importance of Statistical Methods in Industry," General Electric Review, 51:14, December 1948.

in Figure 12, was the only such device for testing 100% sampling which was submitted in response to the writer's circular letter.[3]

Apparatus. Automatic feeding device with slightly inclined inspection table or chute to keep a constant supply of 10 mm beads slowly moving in front of the inspector; light on the inspection area; container for "defective" beads; lot for inspection containing 3000 wooden beads (10 mm diameter), of which 2900 are acceptable (2700 regular and 200 having holes plugged with wooden pins) and 100 are defective (having soft iron pins through the holes); permanent magnet; sampling paddle with 50 holes.

Procedure. The inspector should be allowed 25 minutes to observe all 3000 mixed beads. Upon completion of this inspection, the permanent magnet is passed over the accepted beads. Any defectives overlooked will be attracted to the magnet. Then the magnet is passed over the container holding defective beads. Any beads which are not attracted to it are noted; these are acceptable beads which have been counted as defectives.

The procedure may be continued by either (1) thoroughly returning the defectives into the lot and mixing thoroughly for comparison by another inspector, or (2) taking the accepted lot with the defectives missed still in it as a new lot for a

[3]See Appendix, Tables IX and X.

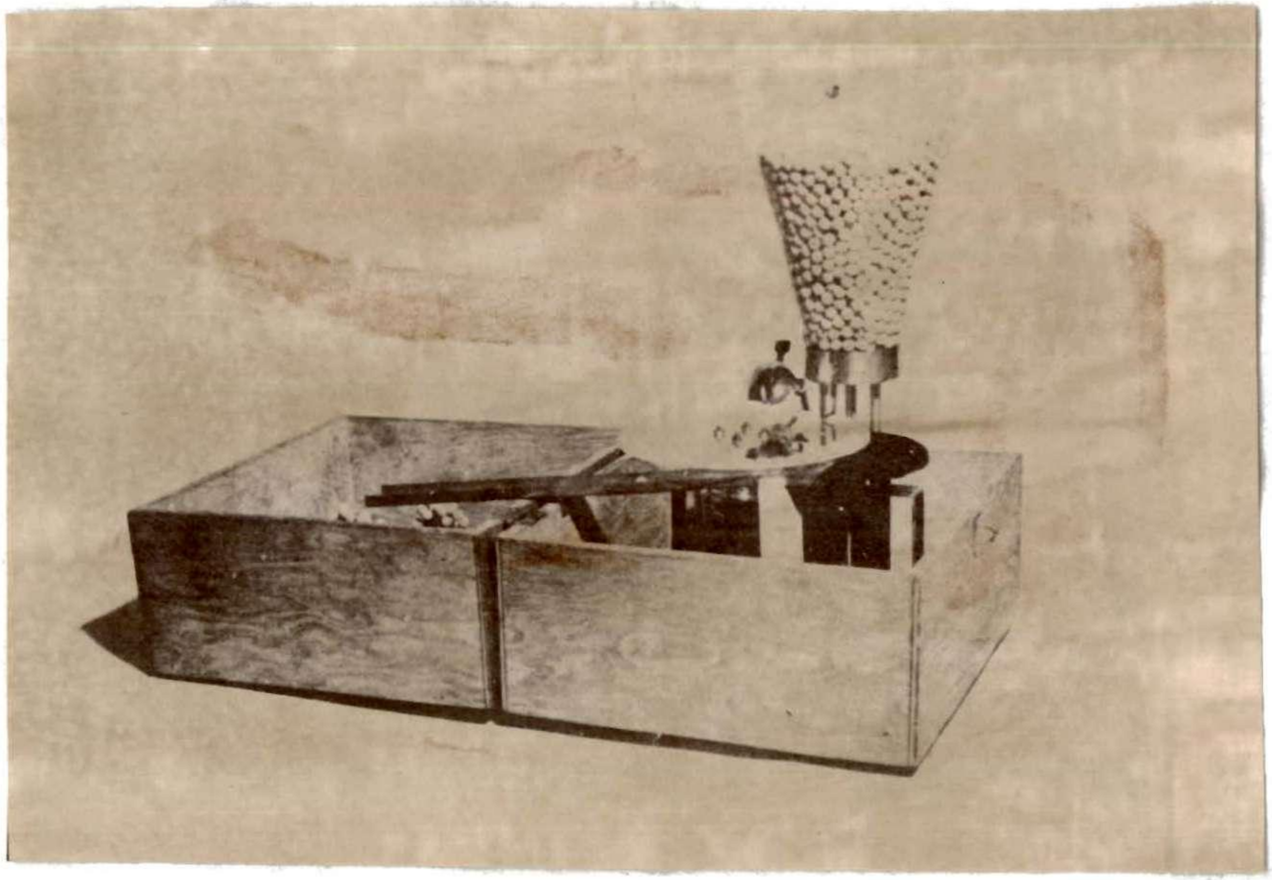

Figure 12

Demonstration Device to show that 100% Inspection is not 100% effective

Photo by Timken Roller Bearing Company, Canton, 6, Ohio.

second inspector to get out these few defectives. (200% Sampling.)

Experience at the Timken Roller Bearing Company using this device has shown an average score of about 83%.

CHAPTER VIII

SINGLE ACCEPTANCE SAMPLING TECHNIQUES

Objective. The objective of this project is to examine some important characteristics of single acceptance sampling techniques, and to compare their effectiveness on the basis of average outgoing quality and total inspection required at different levels.

Discussion. Acceptance sampling depends on mathematical theory which is too involved to be discussed in a presentation of this type.[1] No discussion or vindication of the theory of sampling is herewith attempted, merely a description of some single sampling procedures and their characteristics.

Single sampling involves any procedure wherein one sample forms a basis for acceptance or rejection of the lot from which the sample was taken. The decision is based on the number of defectives permitted in one acceptable sample or "acceptance number," as determined by the level of quality expected for the entire lot. The acceptance numbers increase as the sample size is increased or as the level of acceptable quality is reduced.

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[1] An elementary discussion of sampling procedures with a minimum of mathematical justification is found in C. W. Kennedy, Quality Control Methods (New York: Prentice-Hall, 1948), Part I, pp. 10-87.

Any sampling plan involves a factor called "consumers risk," designated β, which is the risk the buyer is willing to take that lots of worse quality than minimum level will be accepted by the sampling plan.[2] The consumer's safe point is called the lot tolerance percent defective. Another factor, known as the producer's risk,[3] designated α, is the ratio (or percentage) of good lots that will be rejected by the sampling plan. The safe point for the producer is termed the A.Q.L. or acceptable quality level. The measure of any sampling plan is its relative effectiveness in approaching a perfect evaluation of all lots. In a single sampling plan a larger sample from a given lot size has a higher acceptance number and a better characteristic curve, but this plan requires more inspection and hence greater inspection cost.[4] If, however, all rejected lots are 100% detailed, the total number of specimens inspected may be less for the larger samples.

The probability of acceptance of a lot can be estimated by means of the binomial theorem, and in most cases by the Poisson approximation. The sample size (n), the acceptance number (c), and the risks of the producer (α), and of the consumer (β) are so related that if the producer and consumer risks are

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[2] L. H. C. Tippett, Technological Applications of Statistics (New York: John Wiley & Sons, Inc., 1950), p. 62.

[3] Kennedy, op. cit., p. 26.

[4] Eugene L. Grant, Statistical Quality Control (New York: McGraw-Hill Book Company, Inc., 1946), p. 351.

specified, it is possible to find the value of (n) which corresponds to a given value of c, or vice versa.[5]

Some relationship between these factors or the picture of what a sampling plan can be expected to do is best illustrated by a graph called the Operating Characteristic Curve for the sampling plan. Figure 13 represents a plan for a large lot size from which a sample of 177 items is taken at random.[6] If the number of defectives in the sample exceeds the acceptance number ($c = 2$), the entire lot is rejected, but if two or less defectives are found, the lot is passed. The curve indicates that on the average we may expect this sampling plan to pass 74% (Pa) ($\alpha = 1 - Pa = .26$) of all incoming lots which contain 1% defective (p_1) = A.Q.L., but only 10% (β) of lots which contain 3% defectives (p_2) = (LTPD).

The protection offered both as to α and β is affected more by n and c than by the lot size N unless the lot is small. The minimum size lot (N) should be[7] at least 300. Figure 14 illustrates some characteristic curves for 10% sampling plans in which $n/N = 0.10$, for which the protection is commonly misunderstood. This may be contrasted with the series of sampling

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[5]Tippett, op. cit., p. 67.

[6]Niles E. Barnard and John A. Henry, Manual in Statistical Quality Control (Chicago: The Mid-west Quality Control Conference, Box 1097, Chicago, 90, Illinois, 1949), p. 7.

[7]Norbert L. Enrick, Quality Control (New York: The Industrial Press, 1948), p. 6.

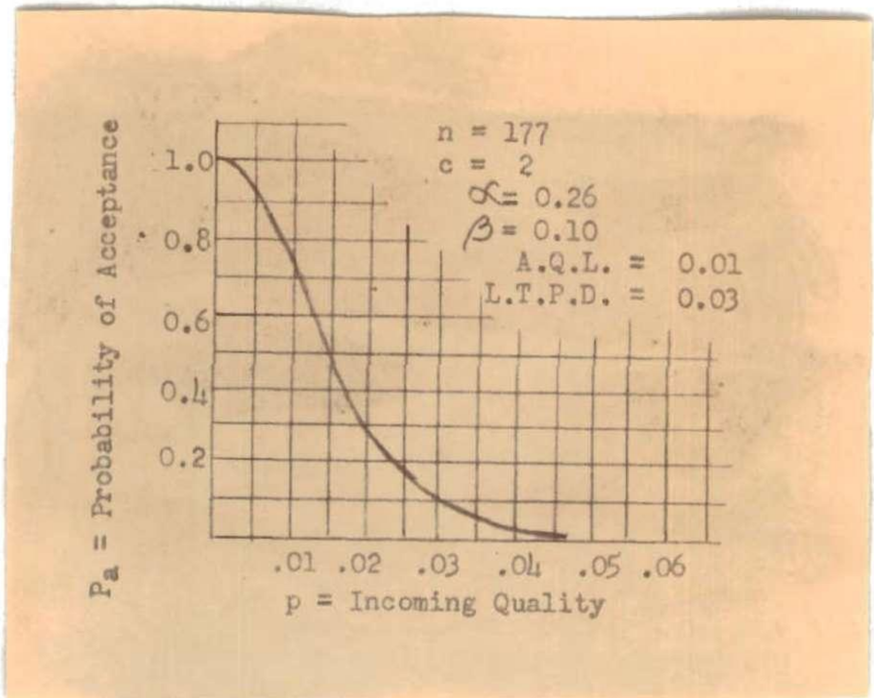

Figure 13

Operating Characteristic Curve

Source: N. E. Barnard and J. A. Henry, Manual in Statistical Quality Control (Chicago: The Midwest Quality Control Conference, Box 1097, Chicago, 90, Illinois, 1949), Figure 8, p. 8.

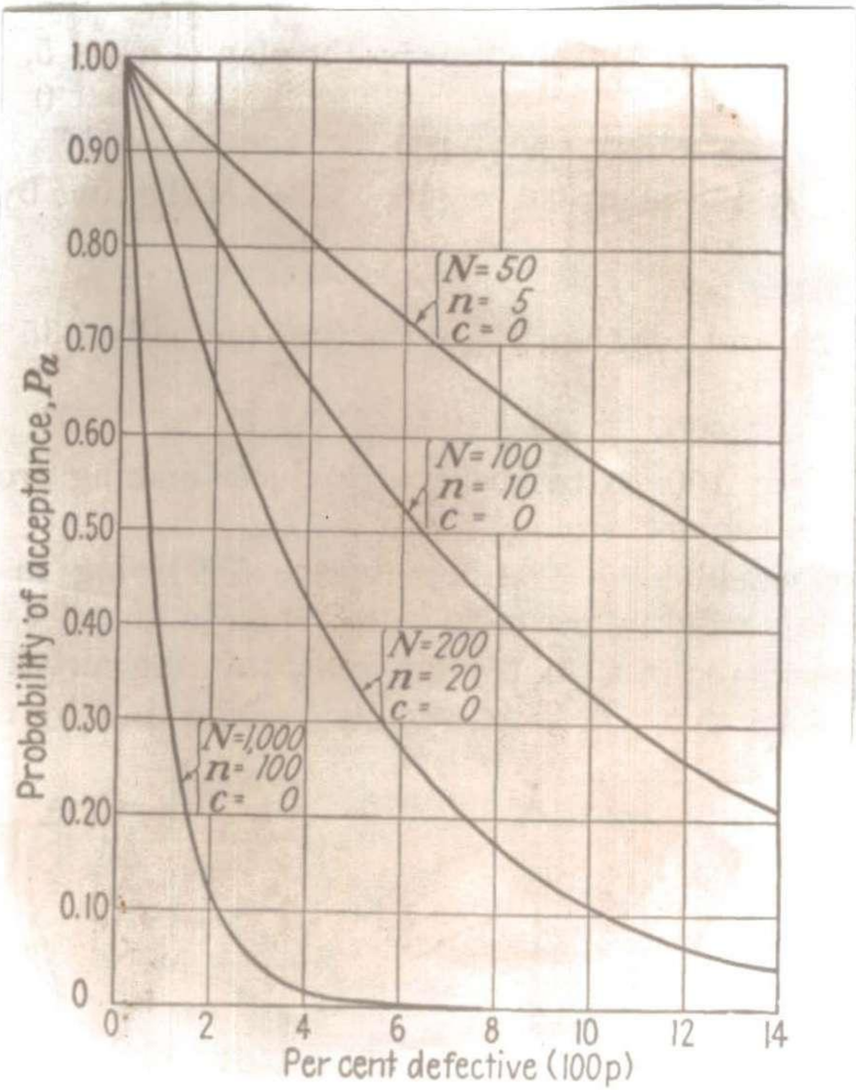

Figure 14

Comparison of Characteristic Curves for four single sampling plans involving 10% Samples

Source: E. L. Grant, <u>Statistical Quality Control</u> (New York: McGraw-Hill Book Company, 1946), p. 343.

plans in Figure 15 based on n = 20, leaving little doubt as to the effect of changing n. By increasing n and allowing c to exceed o, the characteristic curve can be made to approach a vertical line through the A.Q.L., which is the 100% inspection curve.[8]

The problem of sample size should not be thought of as what size sample should be taken but rather as what size sample should be taken to do what.[9] It is possible to design a sampling plan which can accomplish almost any reasonable guarantees, but the more selective the plan, the closer the overall sampling approaches 100% inspection.

For every level of incoming quality there is a corresponding level of outgoing quality which is accepted by a particular sampling plan. While the outgoing quality varies for each accepted sample, an average quality level can be computed from the probability of acceptance, assuming that all rejected lots are 100% detailed (all defectives replaced in the final outgoing total). This establishes the average outgoing quality (A.O.Q.). If perfect lots are submitted, all will be accepted and the outgoing quality will be perfect; on the other hand if all lots are bad, then all will be rejected and submitted to 100% inspection resulting in perfect outgoing quality. Between

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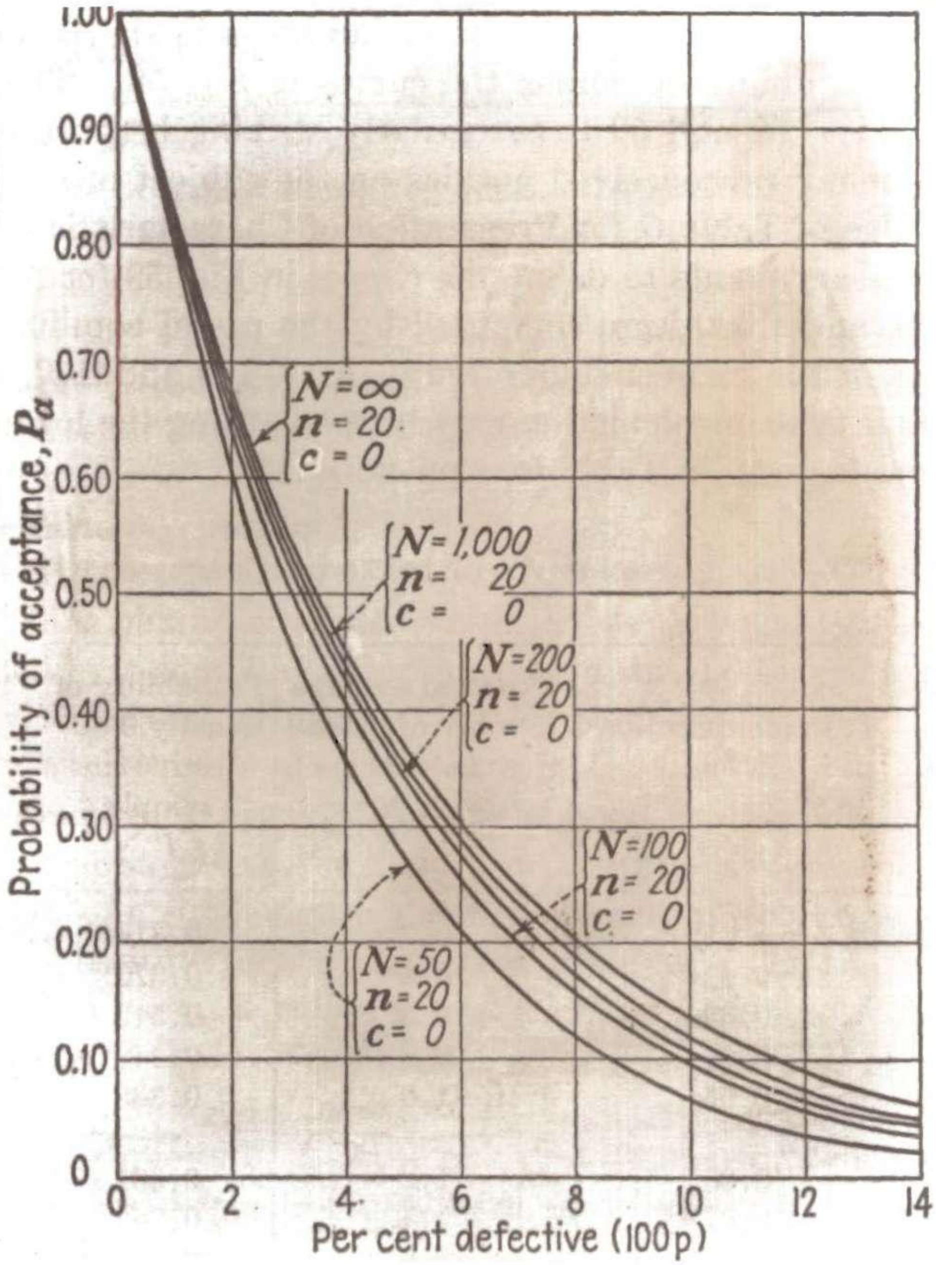
[8]Grant, op. cit., p. 350.

[9]Leslie E. Simon, An Engineers Manual of Statistical Methods (New York: John Wiley & Sons, 1941), p. 84.



Probability of acceptance, P_a
Per cent defective (100p)
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0
2
4
6
8
10
12
14
N = ∞, n = 20, c = 0
N = 1,000, n = 20, c = 0
N = 200, n = 20, c = 0
N = 100, n = 20, c = 0
N = 50, n = 20, c = 0

Figure 15

Comparison of Characteristic Curves for five single sampling plans, each involving Samples of 20 with acceptance number of 0

Source: E. L. Grant, Statistical Quality Control (New York: McGraw-Hill Book Company, 1946), p. 345.

these extremes the average outgoing quality reaches a maximum (called A.O.Q.L. or average outgoing quality limit) which is a convenient measure of the effectiveness of a sampling plan.

The selection of an appropriate single sampling plan has been simplified by the publication of sampling tables, such as those by H. F. Dodge and H. G. Romig, an excerpt from which is shown in Figure 16. It will be noted that the tables are set up on two bases. The SL tables assume a consumer's risk of 10%, and indicate plans for stated values of lot tolerance percent defective. The SA tables are likewise based on a consumer's risk of 10%, but indicate sampling plans for stated values of the average outgoing quality limit.

Analysis of existing techniques. Of the demonstrations submitted to illustrate sampling,[10] the use of colored beads is clearly the most popular. Some special sampling charts are used to show the applications and advantages of sampling techniques. One unique demonstration related to sampling is a machine for attributes illustrated and described in a book by Mr. L. E. Simon.[11] A measured sample of colored beads from a known population is allowed to fill a groove where the sample can be checked, and the percentage defects compared with the p of the lot. Three jars are located immediately below the

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[10] See results of survey in Appendix, Tables IX and X.

[11] Simon, loc. cit., p. 8.

TABLE I CONT'D: SINGLE SAMPLING LOT INSPECTION TABLES—BASED ON STATED VALUES OF "LOT TOLERANCE PER CENT DEFECTIVE" AND CONSUMER'S RISK = 0.10

TABLE SL-4

Lot Tolerance Per Cent Defective = 4.0%

Process Average %	0-.04			.05-.40			.41-.80			.81-1.20			1.21-1.60			1.61-2.00		
Lot Size	n	c	AOQL %	n	c	AOQL %	n	c	AOQL %	n	c	AOQL %	n	c	AOQL %	n	c	AOQL %
1-35	All	0	0	All	0	0	All	0	0	All	0	0	All	0	0	All	0	0
36-50	34	0	.35	34	0	.35	34	0	.35	34	0	.35	34	0	.35	34	0	.35
51-100	44	0	.47	44	0	.47	44	0	.47	44	0	.47	44	0	.47	44	0	.47
101-200	50	0	.55	50	0	.55	50	0	.55	50	0	.55	50	0	.55	50	0	.55
201-300	55	0	.57	55	0	.57	85	1	.71	85	1	.74	85	1	.71	85	1	.71
301-400	55	0	.58	55	0	.58	90	1	.72	120	2	.80	120	2	.80	145	3	.86
401-500	55	0	.60	55	0	.60	90	1	.77	120	2	.87	150	3	.91	150	3	.91
501-600	55	0	.61	95	1	.76	125	2	.87	125	2	.87	155	3	.93	185	4	.95
601-800	55	0	.62	95	1	.78	125	2	.93	160	3	.97	190	4	1.0	220	5	1.0
801-1000	55	0	.63	95	1	.80	130	2	.92	165	3	.98	220	5	1.1	255	6	1.1
1001-2000	55	0	.65	95	1	.84	165	3	1.1	195	4	1.2	255	6	1.3	315	8	1.4
2001-3000	95	1	.86	130	2	1.0	165	3	1.1	230	5	1.3	320	8	1.4	405	11	1.6
3001-4000	95	1	.86	130	2	1.0	195	4	1.2	260	6	1.4	350	9	1.5	465	13	1.6
4001-5000	95	1	.87	130	2	1.0	195	4	1.2	290	7	1.4	380	10	1.6	520	15	1.7
5001-7000	95	1	.87	130	2	1.0	200	4	1.2	290	7	1.5	410	11	1.7	575	17	1.9
7001-10,000	95	1	.88	130	2	1.1	230	5	1.4	325	8	1.5	440	12	1.7	645	19	1.9
10,001-20,000	95	1	.88	165	3	1.2	265	6	1.4	355	9	1.6	500	14	1.8	730	22	2.0
20,001-50,000	95	1	.88	165	3	1.2	295	7	1.5	380	10	1.7	590	17	2.0	870	26	2.1
50,001 100,000	95	1	.88	200	4	1.3	325	8	1.6	410	11	1.8	620	18	2.0	925	29	2.2

TABLE SL-5

Lot Tolerance Per Cent Defective = 5.0%

Process Average %	0-.05			.06-.50			.51-1.00			1.01-1.50			1.51-2.00			2.01-2.50		
Lot Size	n	c	AOQL %	n	c	AOQL %	n	c	AOQL %	n	c	AOQL %	n	c	AOQL %	n	c	AOQL %
1-30	All	0	0	All	0	0	All	0	0	All	0	0	All	0	0	All	0	0
31-50	30	0	.49	30	0	.49	30	0	.49	30	0	.49	30	0	.49	30	0	.49
51-100	37	0	.63	37	0	.63	37	0	.63	37	0	.63	37	0	.63	37	0	.63
101-200	40	0	.74	40	0	.74	40	0	.74	40	0	.74	40	0	.74	40	0	.74
201-300	43	0	.74	43	0	.74	70	1	.92	70	1	.92	95	2	.99	95	2	.99
301-400	44	0	.74	44	0	.74	70	1	.99	100	2	1.0	120	3	1.1	145	4	1.1
401-500	45	0	.75	75	1	.95	100	2	1.1	100	2	1.1	125	3	1.2	150	4	1.2
501-600	45	0	.76	75	1	.98	100	2	1.1	125	3	1.2	150	4	1.3	175	5	1.3
601-800	45	0	.77	75	1	1.0	100	2	1.2	130	3	1.2	175	5	1.4	200	6	1.4
801-1000	45	0	.78	75	1	1.0	105	2	1.2	155	4	1.4	180	5	1.4	225	7	1.5
1001-2000	45	0	.80	75	1	1.0	130	3	1.4	180	5	1.6	230	7	1.7	280	9	1.8
2001-3000	75	1	1.1	105	2	1.3	135	3	1.4	210	6	1.7	280	9	1.9	370	13	2.1
3001-4000	75	1	1.1	105	2	1.3	160	4	1.5	210	6	1.7	305	10	2.0	420	15	2.2
4001-5000	75	1	1.1	105	2	1.3	160	4	1.5	235	7	1.8	330	11	2.0	440	16	2.2
5001-7000	75	1	1.1	105	2	1.3	185	5	1.7	260	8	1.9	350	12	2.2	490	18	2.4
7001-10,000	75	1	1.1	105	2	1.3	185	5	1.7	260	8	1.9	380	13	2.2	535	20	2.5
10,001-20,000	75	1	1.1	135	3	1.4	210	6	1.8	285	9	2.0	425	15	2.3	610	23	2.6
20,001-50,000	75	1	1.1	135	3	1.4	235	7	1.9	305	10	2.1	470	17	2.4	700	27	2.7
50,001-100,000	75	1	1.1	160	4	1.6	235	7	1.9	355	12	2.2	515	19	2.5	770	30	2.8

n = Size of Sample; entry of "All" indicates that each piece in lot is to be inspected.
c = Allowable Defect Number for Sample.
AOQL = Average Outgoing Quality Limit.

TABLE III CONT'D: SINGLE SAMPLING LOT INSPECTION TABLES—BASED ON STATED VALUES OF "AVERAGE OUTGOING QUALITY LIMIT"

TABLE SA-5.0

Average Outgoing Quality Limit = 5.0%

Process Average %	0-.10			.11-1.00			1.01-2.00			2.01-3.00			3.01-4.00			4.01-5.00		
Lot Size	n	c	p_t%	n	c	p_t%	n	c	p_t%	n	c	p_t%	n	c	p_t%	n	c	p_t%
1-5	All	0	-	All	0	-	All	0	-	All	0	-	All	0	-	All	0	-
6-50	6	0	30.5	6	0	30.5	6	0	30.5	6	0	30.5	6	0	30.5	6	0	30.5
51-100	7	0	27.0	7	0	27.0	7	0	27.0	14	1	26.5	14	1	26.5	14	1	26.5
101-200	7	0	27.5	7	0	27.5	16	1	24.0	16	1	24.0	16	1	24.0	24	2	21.5
201-300	7	0	27.5	16	1	24.0	16	1	24.0	16	1	24.0	25	2	21.0	25	2	21.0
301-400	7	0	27.5	16	1	24.0	16	1	24.0	26	2	20.0	26	2	20.0	35	3	18.8
401-500	7	0	27.5	16	1	24.0	16	1	24.0	26	2	20.0	36	3	18.3	46	4	17.0
501-600	7	0	28.0	16	1	24.0	26	2	20.0	26	2	20.0	37	3	17.9	47	4	16.6
601-800	7	0	28.0	16	1	24.0	27	2	19.4	37	3	17.9	48	4	16.3	60	5	15.2
801-1000	7	0	28.0	17	1	22.5	27	2	19.5	37	3	17.9	48	4	16.3	70	6	14.3
1001-2000	7	0	28.0	17	1	23.0	27	2	19.6	38	3	17.6	60	5	15.3	85	7	13.7
2001-3000	7	0	28.0	17	1	23.0	38	3	17.6	50	4	15.8	75	6	13.9	125	10	12.3
3001-4000	17	1	23.0	27	2	19.6	39	3	17.0	60	5	15.4	85	7	13.8	140	11	11.8
4001-5000	17	1	23.0	27	2	19.6	39	3	17.0	65	5	14.2	100	8	12.9	155	12	11.6
5001-7000	17	1	23.0	27	2	19.7	39	3	17.1	75	6	13.9	115	9	12.3	185	14	11.0
7001-10,000	17	1	23.0	27	2	19.7	50	4	15.9	75	6	14.0	130	10	12.0	225	17	10.4
10,001-20,000	17	1	23.0	27	2	19.7	50	4	15.9	90	7	13.1	170	13	11.0	305	22	9.6
20,001-50,000	17	1	23.0	39	3	17.1	65	5	14.3	115	9	12.3	215	16	10.4	400	28	9.0
50,001-100,000	17	1	23.0	39	3	17.1	75	6	14.0	145	11	11.6	275	20	9.8	450	31	8.8

TABLE SA-7.0

Average Outgoing Quality Limit = 7.0%

Process Average %	0-.14			.15-1.40			1.41-2.80			2.81-4.20			4.21-5.60			5.61-7.00		
Lot Size	n	c	p_t%	n	c	p_t%	n	c	p_t%	n	c	p_t%	n	c	p_t%	n	c	p_t%
1-5	All	0	-	All	0	-	All	0	-	All	0	-	All	0	-	All	0	-
6-50	5	0	35.5	5	0	35.5	5	0	35.5	5	0	35.5	5	0	35.5	5	0	35.5
51-100	5	0	36.0	5	0	36.0	5	0	36.0	11	1	28.5	11	1	28.5	11	1	28.5
101-200	5	0	36.5	5	0	36.5	11	1	30.5	11	1	30.5	18	2	26.5	18	2	26.5
201-300	5	0	36.5	12	1	28.5	12	1	28.5	18	2	26.5	18	2	26.5	25	3	26.0
301-400	5	0	37.0	12	1	28.5	12	1	28.5	19	2	25.5	26	3	25.0	33	4	23.5
401-500	5	0	37.0	12	1	28.5*	19	2	25.5	19	2	25.5	26	3	25.0	34	4	23.0
501-600	5	0	37.0	12	1	28.5	19	2	25.5	27	3	24.5	34	4	23.0	42	5	21.5
601-800	5	0	37.0	12	1	29.0	19	2	25.5	27	3	24.5	35	4	22.5	50	6	20.5
801-1000	5	0	37.0	12	1	29.0	19	2	25.5	27	3	24.5	43	5	21.5	60	7	19.3
1001-2000	5	0	37.0	12	1	29.0	27	3	24.5	36	4	22.0	50	6	21.0	70	8	17.7
2001-3000	12	1	29.0	19	2	25.5	28	3	23.5	45	5	20.5	60	7	19.6	100	11	16.5
3001-4000	12	1	29.0	20	2	24.5	28	3	24.0	45	5	20.5	70	8	18.1	120	13	15.8
4001-5000	12	1	29.0	20	2	24.5	36	4	22.0	55	6	19.0	80	9	17.3	140	15	15.1
5001-7000	12	1	29.0	20	2	24.5	36	4	22.0	55	6	19.1	90	10	16.8	160	17	14.6
7001-10,000	12	1	29.0	20	2	24.5	36	4	22.0	65	7	18.4	110	12	15.9	195	20	13.9
10,001-20,000	12	1	29.0	28	3	24.0	45	5	20.5	75	8	17.8	135	14	15.2	240	24	13.2
20,001-50,000	12	1	29.0	28	3	24.0	55	6	19.2	95	10	16.6	175	18	14.1	310	30	12.4
50,001-100,000	12	1	29.0	28	3	24.0	55	6	19.2	115	12	15.8	210	21	13.4	355	34	12.1

n = Size of Sample; entry of "All" indicates that each piece in lot is to be inspected.
c = Allowable Defect Number for Sample.
p_t = Lot Tolerance Per Cent Defective corresponding to a Consumer's Risk (P_C) = 0.10.

Figure 16

Two Types of Single Sampling Inspection Tables

Source: H. F. Dodge and H. G. Romig, <u>Sampling Inspection Tables--Single and Double Sampling</u> (New York: John Wiley & Sons, Inc., 1944).

groove to collect (1) Samples of p greater than p of the lot, (2) samples with p equal to p of the lot, and (3) samples with p less than p of the lot. Tests of the device with various values for p show consistent results of 26%, 39%, and 35% for the samples allotted to the three respective jars, confirming the little understood truth that samples which are better than the p of the lot are found more often than those which are worse than the p of the lot.

Two special pieces of apparatus for expediting the use of colored beads were also uncovered. The simplest device consists of sampling holes drilled into the bottom of a closed box which contains a given population.[12] A transparent section above the holes permits inspection of the sample, which can be made by merely tipping the box until all the population flows over the holes and rolls back, leaving the sample below the window in the built-in paddle. Another sampling machine, designed by T. H. Brown and D. H. Leavens, consists of a mixing container, similar to an oil can, in which a sample can be obtained by tilting the can so that the beads roll into its transparent spout.[13]

Of these devices, the use of colored beads is unchallenged as a means of demonstrating the principles of sampling.

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[12]Suggested by Professor S. S. Wilks, Princeton University, personal letter of July 24, 1950.

[13]E. G. Olds and L. A. Knowler, "Teaching Statistical Quality Control for Town and Gown," American Statistical Association Journal, 44:229, June 1949.

The sampling machine devised by Mr. Simon is especially constructed to illustrate a particular aspect of sampling and is not readily adapted to demonstrations of other characteristics of single, double, or sequential sampling.

The other two sampling machines mentioned above merely speed up the handling of colored beads by using special sampling devices in place of a hand paddle. No analytical table has been compiled for the comparison of demonstrations of sampling because of the outstanding superiority of the use of colored beads.

Apparatus. Colored beads--1000 white, 240 colored; 50 hole and 5 hole sampling paddles; 2 bowls; several small boxes or containers for holding lots; observation data sheet.

Procedure. An 8% population of 1000 beads (920 white and 80 red) is prepared, mixed thoroughly, and divided into lots of 50 (N = 50). From each lot draw a sample of (n = 5) using acceptance number of c = 0. If no red balls appear in the sample, the whole lot is accepted and poured into the bowl for accepted lots. If any red balls appear, the lot is rejected, and all red beads are removed to the bowl of rejects, but the white beads are poured into the bowl with the accepted lots. When the 20 lots have been checked, the total number of defectives in the reject bowl is subtracted from the total number of defectives to determine the defectives in the accepted bowl. This forms a basis for computing the A.O.Q. at this level.

The large population is changed to 12% defective by replacing 40 white beads with 40 red beads, and the procedure described above is repeated, using the same sampling plan.

The entire test with 16%, 20%, and 24% populations is repeated, testing each with the same sampling plan, and submitting all rejected lots to 100% inspection before acceptance.

The probable percentage of samples which should be accepted is computed for each population by using Poisson tables, and the operating Characteristic curves are drawn through these five points.

The actual and theoretical acceptance of lots are compared.

The A.O.Q. for each population is computed and compared with the observed A.O.Q. for each.

CHAPTER IX

DOUBLE AND SEQUENTIAL ACCEPTANCE SAMPLING TECHNIQUES

Objective. The objective of this project is to examine and compare some important characteristics of double and sequential acceptance sampling techniques.

Discussion. Double acceptance sampling is a device for reducing inspection required to reach the decision to accept or reject a lot. The plan is based on the preface that very good and very poor lots can be readily detected with a much smaller sample than would be required for a single sample acceptance plan. For those lots which are near the acceptance level, closer measurement is required, which is the purpose of the second sample. It must be clearly understood that double sampling is not just taking two samples instead of one, nor is it splitting a sample into two parts and averaging the results.[1] One other common misunderstanding is that it is "giving another chance" to those lots which are rejected by a single sample.

The acceptance of a lot by the first sample is based on a small acceptance number of defectives in the sample which will allow only a very good lot to pass. This acceptance number is usually denoted as c_1. The number of defectives in the first

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[1]Clifford W. Kennedy, Quality Control Methods (New York: Prentice-Hall, Inc., 1948).

sample is also compared to a maximum limit c_2 above which the lot is rejected as being very poor. Those samples which have defectives between c_1 and c_2 require an additional sample to determine the final disposition of the lot. The second sample is selected of such a size that the total defectives in the combined samples can again be compared with the c_2 limit. If the total defectives in the two samples exceeds the c_2 limit the lot is rejected, otherwise it is accepted.[2] This procedure is illustrated in Figure 17 which is a double sampling table derived from data used by the U. S. Army in sampling war-time production. This is a slight difference of procedure, in that Figure 17 requires rejection if the total number of defectives equal or exceed the c_2 limit.

Double acceptance sampling plans can be set forth graphically in characteristic curves, and the A.O.Q., and A.O.Q.L. can be determined as in the single sample procedures described in Chapter VIII.

As in the case of single sampling it is possible to "tailor make" a double sampling plan to meet the requirements of a given situation. The Dodge Romig tables[3] include two sets of double sampling tables. One set is based on lot tolerance

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[2] L. H. C. Tippett, Technological Applications of Statistics (New York: John Wiley and Sons, 1950), p. 72.

[3] H. F. Dodge and H. G. Romig, Sampling Inspection Tables (New York: John Wiley and Sons, 1944).

Batch Size →	50-100	101-200	201-500	501-1000	1000-5000	5000-10,000	Over 10,000
Take 1st sample of →	20	25	40	60	125	200	300
If defects in 1st sample are equal to or less than → accept lot	0	0	1	2	3	6	8
If defects in 1st sample exceed → reject lot	2	3	5	6	10	16	20
If defects in 1st sample are → take 2d sample	1	1 or 2	2 to 4	3 to 5	4 to 9	7 to 15	9 to 19
2d sample size →	20	30	80	110	185	300	400
If defects in 2d sample added to defects in 1st sample are less than → accept lot	2	3	5	6	10	16	20
If defects in 2d sample added to defects in 1st sample equal or exceed → reject lot	2	3	5	6	10	16	20

Figure 17

Double Sampling Table. (2% limit of defective work expected (AQL); 5% producer's and consumer's risk, and Upper Limit of Accepted Lots 5% defective.)

Source: C. W. Kennedy, "What is Quality Control," (unpublished address before Conference on Statistical Quality Control at The University of Tennessee, Knoxville, April 13, 1950).

percent defective for which values of n_1, c_1, n_2, $(n_1 + n_2)$ and c_2 are tabulated by process averages and lot sizes. This table also indicates the percent AOQL which can be expected for each plan. The other set of tables is based on values of A.O.Q.L. (average outgoing quality limit). For any given A.O.Q.L., the n_1, c_1, n_2, $(n_1 - n_2)$ and c_2 for various lot sizes are tabulated according to percentage of process average. This table also has a column showing lot tolerance percent defective corresponding to a given consumer's risk ($P_c = 0.10$) for each sampling plan.

The selection of an appropriate double sampling scheme depends upon the desired characteristics, which are measured primarily by the factors which can be found in these and other tables.[4] To get a good idea of how the plan works, however, it often helps to plot an operating characteristic curve for the plan similar to those previously illustrated by Figures 13 and 14.

It is interesting to note that the two most important practical advantages of double sampling are not statistical but psychological. One is that borderline lots are given a "second chance" to be accepted, and the other is that no lot is rejected because of a single defective article.[5]

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[4]Another excellent set of tables for single, double and sequential sampling and a selective basis for choice between them is published by The Statistical Research Group of Columbia University entitled, Sampling Inspection. See Bibliography.

[5]Eugene L. Grant, Statistical Quality Control (New York: McGraw-Hill Book Company, Inc., 1946), p. 376.

Sequential sampling is an extension of the double sampling method to prolong the decision in regard to the borderline samples. The terms, values and conditions used, are the same as those considered in single and double sampling. The plan can be built around a table similar to those shown on Figure 18. It will be noted that on Figure 18 values of n in Table A accumulate by constant values of 39 after the first sample, while in Tables B and C of this Figure, n increases in successive increments of 17. All these tables are independent of lot size, and none of them has an upper limit indicated at which the decision must be either to accept or to reject. This has been the basis for one of the most critical complaints against sequential sampling, in that inspectors often prefer a plan "which can make up its mind."[6] These tables have no lot size indicated and consequently there is no limit to the number of samples which can be taken, as long as the accumulated defective ratio remains between the acceptance limits.

Perhaps the clearest illustration of what a sequential acceptance sampling plan is, and how it works, is a working graph of the plan showing the accumulated defectives plotted against the number of specimens (see Figure 19).

The slope of the limit lines (C_1 and C_2) shown on Figure 19 and their points of intersection with the vertical

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[6]Grant, _op_. _cit_., p. 381.

TABLE I

This table is to be used for critical characteristics only. Unless the lot quality is one of the order of 99% to 100% good, the use of this table will usually result in 100% inspection.

n	Acceptance Number	Rejection Number
55	0	3
94	1	4
133	2	5
172	3	6
211	4	7
250	5	8
289	6	9
328	7	10
367	8	11
406	9	12
445	10	13
384	11	14
523	12	15

TABLE II

This table is to be used for most items. It is designed to pass lots which are not more than 3% defective.

n	Acceptance Number	Rejection Number
31	0	4
48	1	5
65	2	6
82	3	7
99	4	8
116	5	9
133	6	10
150	7	11
167	8	12
184	9	13
201	10	14
218	11	15
235	12	16
252	13	17
269	14	18
286	15	19
303	16	20

TABLE III

This table is to be used where judgment, based on prior knowledge of the particular job, indicates that a reduced sampling scheme may be used.

n	Acceptance Number	Rejection Number
9	0	2
26	1	3
43	2	4
60	3	5
77	4	6
94	5	7
111	6	8
128	7	9
145	8	10
162	9	11
179	10	12
196	11	13

Figure 18

Sequential Sampling Tables

Source: Bausch and Lomb, Inc.

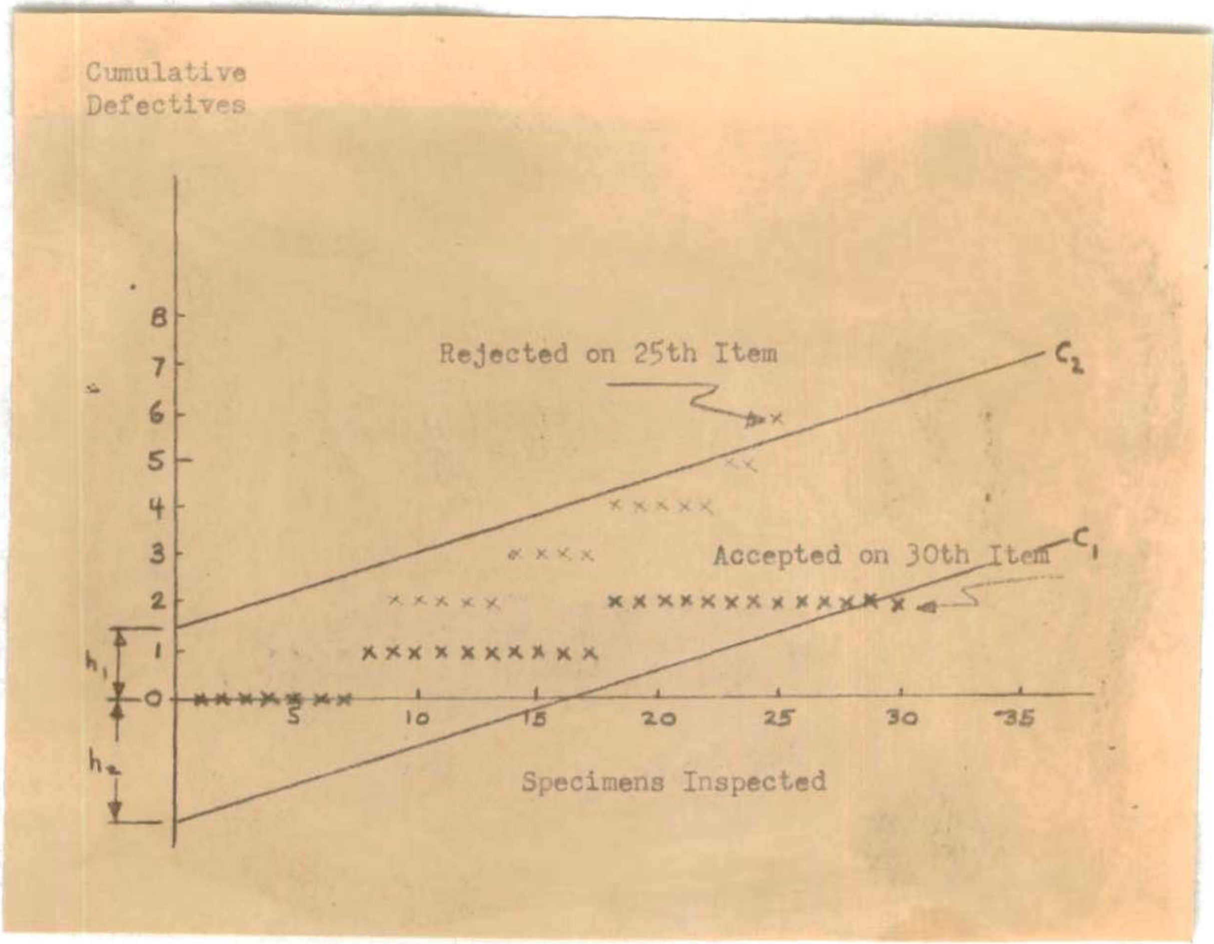

Figure 19

Working Chart for Sequential Sampling Plan axis (h_1 and h_2) can be readily computed from predetermined values of p_1, p_2, α, and β.[7] Once these lines are plotted on the chart, the inspector merely records for each trial (specimen) the cumulative number of defectives by a mark on the graph. If the plotted marks cross the lower limit line (c_1) at any point, the lot is accepted, and conversely the lot is rejected as soon as the plotted values cross the upper (c_2) limit line.

When the lot size is known, the plan can be revised to bring the c_1 and c_2 lines together at a point beyond which further sampling is considered unnecessary.

The tables of acceptance numbers and sample sizes are quite usable by the inspector, who must be instructed in the proper application of these limits. In some cases special devices are constructed to assist the inspector, such as a slide rule which automatically indicates the decision for each setting of defective total against sample size,[8] for a given sequential acceptance sampling plan.

Analysis of existing techniques. The discussion of existing demonstration methods included in Chapter VIII applies also to double and sequential acceptance sampling procedures.

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[7]A detailed explanation of this computation is found in Kennedy, op. cit., pp. 57-65.

[8]E. G. Olds and L. A. Knowler, "Teaching Statistical Quality Control for Town and Gown," American Statistical Association Journal, 44:228, June 1949.

Apparatus. Colored beads (100 red and 1000 white) identical except for color; sampling paddles to hold 50 and 5 beads respectively; two mixing bowls; several (six) small boxes for holding samples; forms for recording data.

Procedure. A population of 1500 colored beads is prepared of which 4% or 60 beads are of a different color and are called defectives.[9] At least 20 samples for each of 4 sampling plans outlined in Table VII are drawn and inspected. Using these plans, samples are drawn to determine if the lot is accepted or rejected. Then the beads are replaced and thoroughly mixed before a new sample is taken. The average percentage of the lots accepted by each plan is compared with the theoretical percent of acceptance, and the total specimens inspected are counted to compare the relative amount of inspection for each plan.

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[9] Procedure from Timken Roller Bearing Company demonstration.

TABLE VII

SAMPLING PLANS TO BE COMPARED

Type of Plan	Sample	Sample Size	Acceptance Number	Rejection Number	Ave. % of Actual Lots Accepted	Theoretical Percent of Acceptance
10%	1st	150	6	7		62
Single	1st	115	8	9		95
Double	1st	75	5	12		
	2nd	150	11	12		95
	1st	30	0	4		
	2nd	30	3	7		
	3rd	30	5	9		
Sequential	4th	30	7	11		95
	5th	30	9	13		
	6th	30	12	15		
	7th	30	14	15		

Source: The Timken Roller Bearing Company.

CHAPTER X

TOLERANCE LIMITS FOR COMBINED PARTS

Objective. The object of this demonstration is to illustrate relationships between tolerance limits for component or mated parts whose manufacture is in a state of statistical control.

Discussion. To the engineer without statistical training it seems perfectly logical to assign tolerances which "add up right." Where two or more parts are assembled, it is common practice under this policy to assign tolerances in such a way that if all components were of minimum size the sum would not be less than the overall tolerance, and conversely that the sum of the maximum values would not exceed the maximum overall tolerance.

If there is only a two piece combination this means roughly one-half the overall tolerance limit can be assigned to each piece. The reduced tolerance may work a hardship in extra precision required. If the tolerances were set equal on a basis that the parts can be produced with equal standard deviation, the general formula can be simplified.[1]

$$\sigma_{sum} = \sqrt{\sigma_1^{\ 2} + \sigma_2^{\ 2}} = \sqrt{2\,\sigma^2_{part}}$$

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[1]Eugene L. Grant, Statistical Quality Control (New York: McGraw-Hill Book Company, Inc., 1946), p. 326.

from which $\sigma_{part} = \sqrt{\frac{\sigma^2_{sum}}{2}}$

Assuming the tolerances are proportional to σ :

Part Tolerance = $(X_{max} - X_{min})$ part = $\sqrt{\frac{(X_{max} - X_{min})^2_{sum}}{2}}$

For example, it may be assumed that the tolerance between the maximum and minimum sum for the assembly of two parts is 0.01 inches. The tolerance using the original basis set forth above would only permit a variation of 0.005 inches for each of the two parts. Using the equation above, however, the tolerance for each part should be $\sqrt{\frac{(.01)^2}{2}}$ or .007 +. As the number of component parts increases, the advantage of the latter method requiring less precision becomes more apparent. It may be assumed that the total of 5 parts must not vary more than $\pm$ 0.01 inches. If the tolerances are assumed equal and proportional to the σ values, the tolerance for each part should be $\sqrt{\frac{(.01)^2}{5}}$ = 0.0044 each instead of $\frac{.01}{5}$ = .002.

While it might appear that 5 x 0.0044 = 0.022" allows more than twice the permissible tolerance, this is a remote possibility. Assuming that 5% of each type part is of the maximum dimension, the probability of any 2 of the maximum size for each part being combined in one assembly is $(.05)^2$ or .0025 which is one-fourth of one percent, and the probability of having all 5 parts in any one assembly come from the upper 5% of each type is $(.05)^5$ = .0000003125 or 3 chances in 10 million.

Another useful relationship is that mating parts requiring a difference of tolerances (where one part fits inside another) use the same formula as for the sum:[2]

$$\sigma_{1,2} = \sqrt{(\sigma_1)^2 + (\sigma_2)^2}$$

Some quality control authorities, such as J. L. Cowles, of Cummins Engine Company, have abolished the use of "No-Go" gages in their plants in order to utilize the full tolerance given on drawings to increase tool life and to eliminate frequent tool adjustments.[3] This is a relatively new viewpoint toward tolerances, but Mr. Cowles points out that it is only another application of the principle of tolerances for combined or mating parts.

<u>Analysis of existing techniques</u>. In response to the writer's circular letter[4] only three types of demonstration were submitted to show this principle for the sum of tolerances on parts or for mating part tolerances. The most common is a series of blocks, dowels or pipe to represent five parts whose combined length is an assembly. Each of the five parts is duplicated with exaggerated variations in dimension following a normal distribution pattern about the nominal dimension.

By selecting one of these parts at random from each of the five types of parts and setting them against a prepared

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[2]Grant, *loc. cit.*

[3]J. L. Cowles, Cummins Engine Company, Inc., Columbus, Indiana. Personal letter, 8/20/50.

[4]Results of survey tabulated in Appendix, Tables IX and X.

background, the sum of their lengths can be compared with the computed maximum and minimum values and the $\pm$ 3 sigma limits, as shown in Figure 20.

A second demonstration of this principle is to use a plug and ring set of certified size, in which the plug is slightly larger than the inside diameter of the ring (as much as 60 millionths). By using heavy oil, the plug can be inserted in the ring and quickly removed by hand.

The third demonstration suggests the use of charts which show the probability of matching parts at the extreme dimensions; a frequency curve of parts made by working to the high side of an OD tolerance compared with a curve of the mated part working to the low side of an ID tolerance; frequency curve for assemblies near desired tolerances when OD and ID are worked through their entire range; and distribution of parts for mated parts where a minimum clearance is essential.

The use of prepared blocks to represent parts is by far the most popular method and is outstanding as compared with the other methods using the appraisal factors shown on Table VIII.

Apparatus. Set of 100 blocks cut to lengths shown in Figure 21; scale for measuring sum of blocks; sheets for computation and recording data; back board for indicating limits as shown in Figure 22.

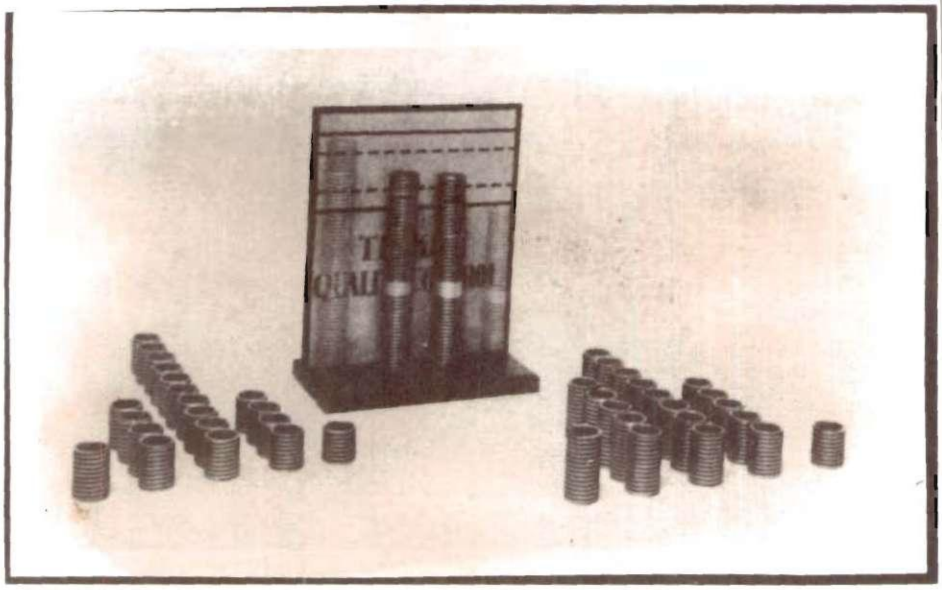

Figure 20

Assembly Tolerances

In the design and manufacture of an assembly certain over-all tolerances must be held. These tolerances are usually divided among the component parts in such a manner that an assembly made entirely of parts of minimum size (or of maximum size) will be OK. This means that the sum of tolerances for the individual parts is equal to that of the assembly.

This demonstration shows that when component parts are made by a quality controlled process and are taken at random for assembly, the sum of component tolerances is much larger than necessary for the great majority of assemblies.

The equipment consists of 100 blocks (or sections of pipe as illustrated above) in five groups of twenty each painted different colors. The lengths and numbers of pieces are shown on Figure 21.

Photo by Timken Roller Bearing Company, Canton 6, Ohio.

TABLE VIII

APPRAISAL OF DEVICES FOR DEMONSTRATING
THAT THE SUM OF TOLERANCES FOR INDIVIDUAL PARTS IS GREATER
THAN THE TOLERANCE OF THE ASSEMBLY

Demonstration	P	T	I	S	C	Total
Colored Blocks	12	10	8	0	2	32
Prepared Ring and Plug	6	10	0	3	0	19
Charts	12	10	0	0	4	26

Factors	Symbol	Maximum Weight
Demonstration of Principle	P	12
Required Demonstration Time	T	10
Adaptability to Individual Participation	I	8
Similarity to Shop Practice	S	6
Economy of Cost of Equipment or Materials	C	4
Total of Five Factors		40

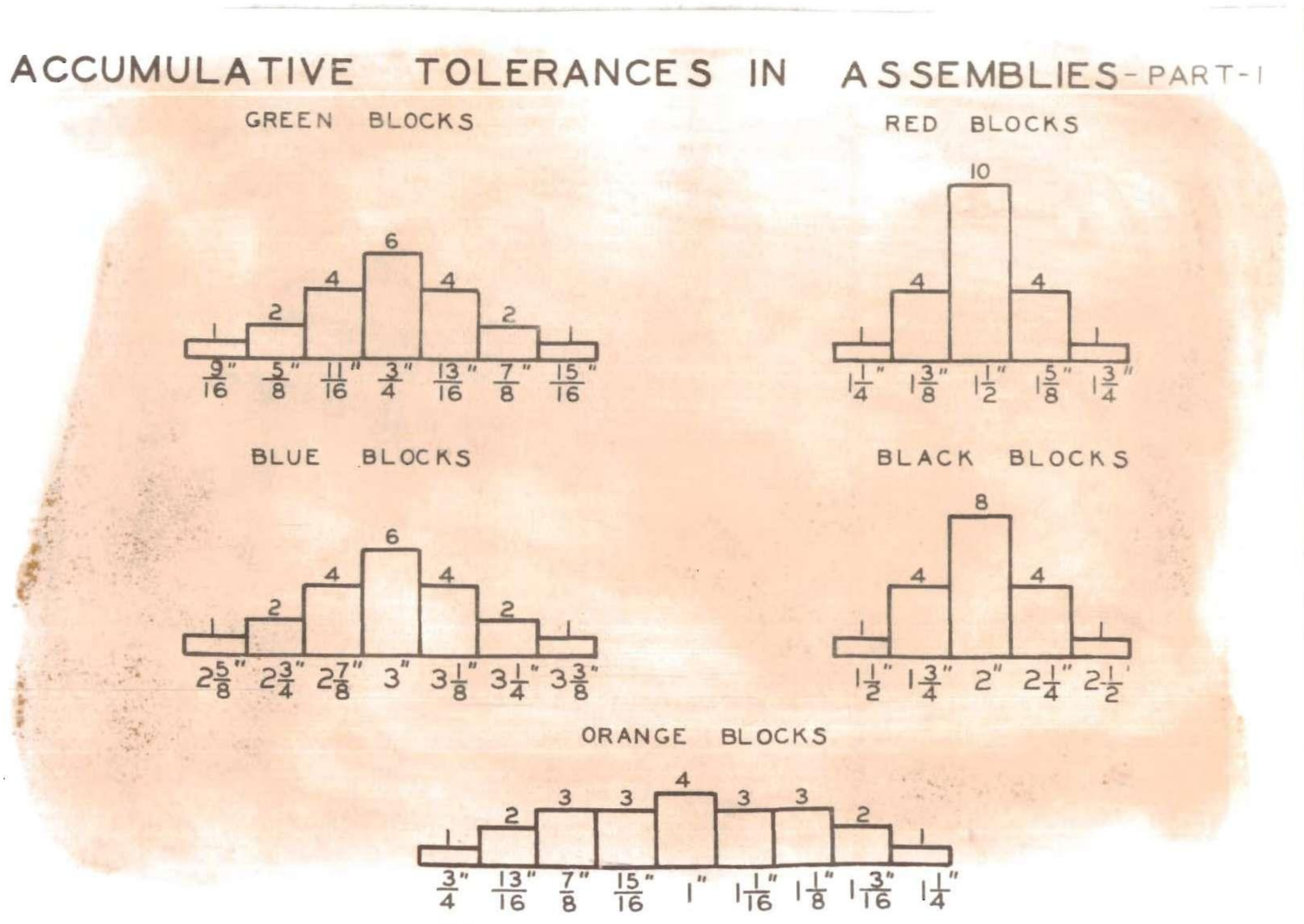

Figure 21

Accumulative Tolerances in Assemblies--Recommended Sizes for Demonstration Blocks

Chart by Professor J. W. Enell, New York University, letter, Aug. 29, 1950

ACCUMULATIVE TOLERANCES IN ASSEMBLIES - PART - 2

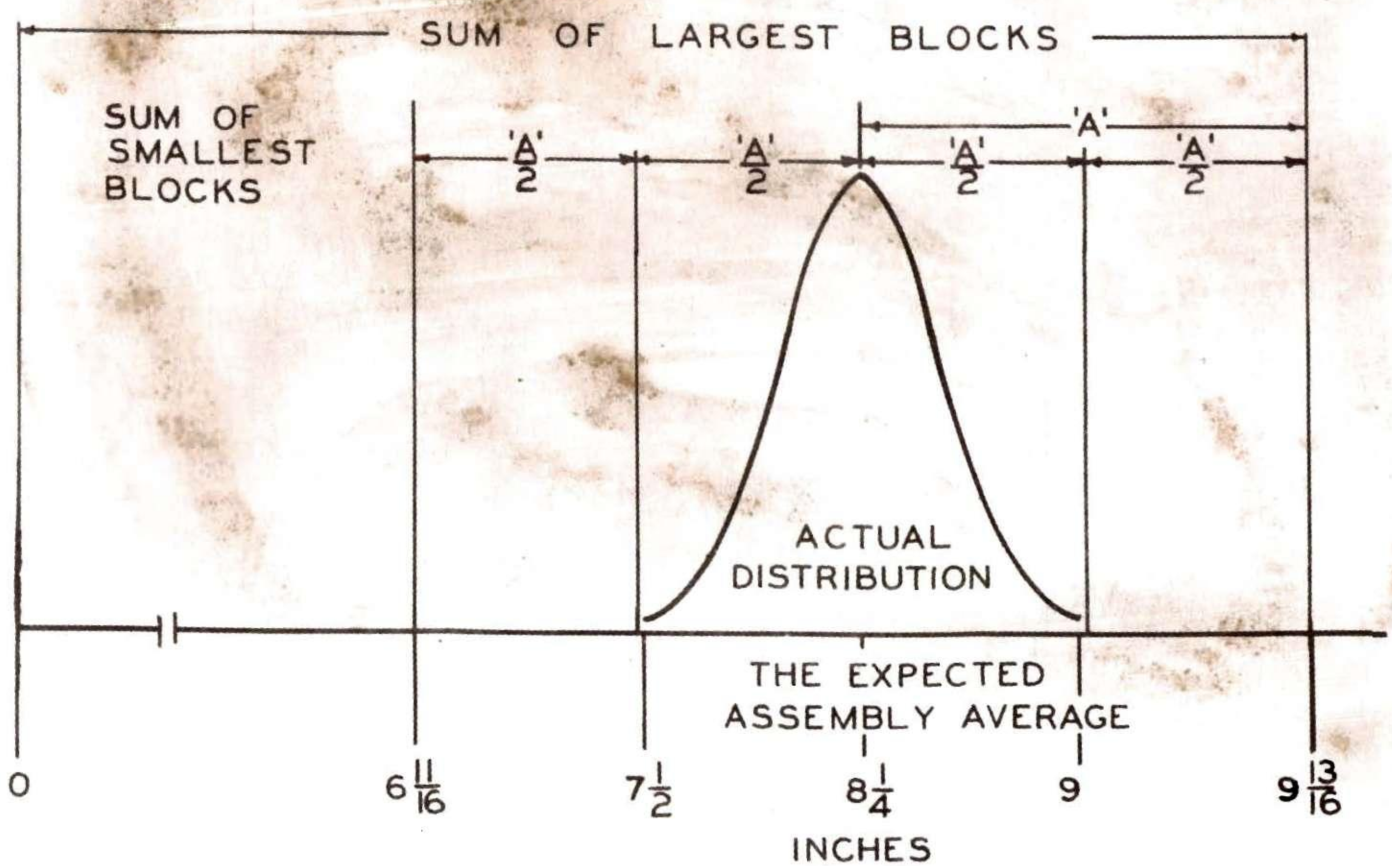

Figure 22

Accumulative Tolerances in Assemblies--Values for Layout of Display Board Limits

Chart by Professor J. W. Enell, New York University, letter, Aug. 29, 1950

<u>Procedure</u>. Using the values for unit lengths of blocks shown in Figure 21 the following is computed:

(a) Length representing sum of five smallest blocks;

(b) Length representing sum of five largest blocks;

(c) Expected assembly average;

(d) Three sigma limits for expected assembly lengths.

The blocks are segregated so that each color is in a separate bin.

One block is drawn at random from each bin and assembled. The combined length is measured with scale and recorded as specimen one. Blocks are replaced in their respective bins, mixed well, and a second sample is drawn as before. Total is recorded as specimen two. This procedure is repeated until at least 50 samples have been drawn. $\overline{X}$ and $\pm 3\sigma$ are computed and compared with actual results and with maximum and minimum values previously computed. Figure 22 may be used to prepare lines on a back board for a quick demonstration without the computations.

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APPENDIX

TABLE IX

SUMMARY OF RESPONSE TO QUESTIONNAIRE[a]

Reference Number	Organizations	Respondent	Data or Suggestions Received
	Industries[b]		
1.	Allis Chalmers Mfg. Co.		
2.	Allison Division, G.M.C.	C. V. Garrett	
3.	Aluminum Co. of America	W. P. Goepfert	
4.	American Bosch Corp.	P. E. Thorpe	
5.	Amplex Mfg. Co. (Chrysler Corp.)		
6.	Anaconda Wire & Cable Co.	H. E. Thompson	x
7.	Bausch & Lomb Optical Co.		
8.	Bell Telephone Laboratories	H. F. Dodge	x
9.	Berrettoni & Associates	J. N. Berrettoni	
10.	Bigelow-Sanford Carpet Company, Inc.	A. G. Klock	
11.	Borg-Warner Corp.		
12.	Brown & Sharpe Mfg. Co.	H. E. Martin	
13.	Brunswick-Balke-Collender Company		
14.	Burgess Batter Co.	G. E. Girard	
15.	Cummins Engine Co., Inc.	J. L. Cowles	x
16.	Dearborn Motors Corp.		
17.	Deere and Company	E. L. Fay	x

TABLE IX

SUMMARY OF RESPONSE TO QUESTIONNAIRE[a] (CONTINUED)

Reference Number	Organizations	Respondent	Data or Suggestions Received
18.	DeLavall Separator Co.		
19.	Delco-Remy Division, G.M.C.	A. Bender, Jr.	x
20.	Detroit Diesel Engine Div., G.M.C.	J. G. Vanhoy	
21.	Don, Edward and Co.		
22.	DuMont, Allen B. Laboratories, Inc.		
23.	DuPont, E. I., de Nemours & Co.	P. D. Deans	x
24.	Engineers Specialty Div.		
25.	Federal Products Corp.	C. W. Kennedy	
26.	Fisk Tire Division	W. A. Egan, Jr.	
27.	Ford Motor Company	R. Smith	x
28.	Fuller Brush Company	A. G. Mason	
29.	General Electric Co.	J. D. Vallier	x
30.	Gillette Safety Razor Co.		
31.	Guide Lamp Div., G.M.C.	E. E. Smith	
32.	Harrison Radiator Division, G.M.C.		
33.	Hamilton Standard Propellors	D. Shainin	x
34.	Hercules Powder Co., Inc.	C. A. Bicking	x
35.	Industrial Manufacturing Corp.		

TABLE IX

SUMMARY OF RESPONSE TO QUESTIONNAIRE[a] (CONTINUED)

Reference Number	Organizations	Respondent	Data or Suggestions Received
36.	International Business Machines Corp.		
37.	International Harvester Co.		
38.	Johns-Manville Corp.	S. Collier	
39.	Johnson Gage Co.	C. W. Johnson	
40.	Johnson and Johnson		
41.	Kimberly-Clark Corp.	C. E. Noble	x
42.	King-Seely Corp.		
43.	Ladish Company	R. L. Hermann	x
44.	Management Controls, Inc.		
45.	Martin, Glenn L. Co.	J. G. Rutherford	
46.	McNally-Pittsburgh Mfg. Corp.	W. K. McAleer	x
47.	Minneapolis-Honeywell Regulator Co.	D. J. Greb	
48.	Minneapolis-Moline Co.	C. V. Slathar	x
49.	Minnesota Mining & Mfg. Co.	J. N. Linnerooth	x
50.	Nash Motors Division		
51.	New Departure Division, G.M.C.	R. E. Young	
52.	New Holland Machine Co.		
53.	Norton Company		
54.	Ohio Rubber Co., The		

TABLE IX

SUMMARY OF RESPONSE TO QUESTIONNAIRE[a] (CONTINUED)

Reference Number	Organizations	Respondent	Data or Suggestions Received
55.	Oldsmobile Division, G.M.C.		
56.	Pratt and Whitney	A. J. F.	
57.	Republic Steel Corp.		
58.	Scintilla Magneto Div.		
59.	Scott and Williams, Inc.	H. D. Edgerly	x
60.	Screw Machine Products		
61.	Sheffield Corp., The	W. I. Wilt	
62.	Smith, A. O. Corp.	L. S. Eichelberger	x
63.	Solar Aircraft Co.		
64.	Standard Gage., Inc.		
65.	Sylvania Electric Products	J. R. Steen	x
66.	Tennessee Eastman Corp.	T. R. Bainbridge	x
67.	Timkin-Detroit Axle Co.	W. S. Oliver	x
68.	Timkin Roller Bearing Co.	R. E. Wagenhals	x
69.	Veeder-Root, Inc.	L. J. Dunn	
70.	Zenith Radio Corp.		
	Engineering Colleges[c]		
71.	Alabama Polytechnic Institute		
72.	Alabama, University of	G. C. K. Johnson	x

TABLE IX

SUMMARY OF RESPONSE TO QUESTIONNAIRE[a] (CONTINUED)

Reference Number	Organizations	Respondent	Data or Suggestions Received
73.	Brooklyn Polytechnic Institute		
74.	Carnegie Institute of Technology	E. G. Olds	x
75.	Case School of Applied Science		
76.	Clarkson College of Technology		
77.	Colorado, University of	J. F. Wagner	x
78.	Columbia University		
79.	Connecticut, University of		
80.	Cornell University	A. Schultz, Jr.	x
81.	Harvard Business School		
82.	Illinois Institute of Technology	W. G. Ireson	
83.	Illinois, University of	J. A. Henry	x
84.	Iowa, University of	L. A. Knowler	x
85.	Johns Hopkins University		
86.	Kansas, University of	M. E. Fesslor	
87.	Lehigh University		
88.	Massachusetts Institute of Technology		

TABLE IX

SUMMARY OF RESPONSE TO QUESTIONNAIRE[a] (CONTINUED)

Reference Number	Organizations	Respondent	Data or Suggestions Received
89.	Michigan State College	W. D. Baten	
90.	Michigan, University of	C. C. Craig	
91.	Minnesota, University of	J. L. Imhoff	
92.	Montana State College		
93.	Newark College of Engineering		
94.	New Mexico College of A. & M.	W. P. Heinzman	
95.	New York University	J. W. Enell	x
96.	Notre Dame, University of		
97.	North Carolina State College	P. Peach	
98.	Northwestern University	M. E. Wescott	x
99.	Ohio State University	L. G. Miller	
100.	Pennsylvania State College	C. E. Bullinger	x
101.	Pennsylvania, University of		
102.	Pittsburgh, University of		
103.	Princeton University	S. S. Wilks	x
104.	Purdue University	I. W. Burr	x
105.	Southern California, University of	L. R. Guild	
106.	Stanford University	E. L. Grant	

TABLE IX

SUMMARY OF RESPONSE TO QUESTIONNAIRE[a] (CONTINUED)

Reference Number	Organizations	Respondent	Data or Suggestions Received
107.	Syracuse University	C. R. Hicks	
108.	Tennessee, University of	R. M. LaForge	x
109.	Texas, A and M College of	A. R. Burgess	x
110.	Virginia Polytechnic Institute	C. A. Horst	
111.	Washington, University of		
112.	West Virginia, University of		
113.	Wisconsin, University of		
114.	Worcester Polytechnic Institute	G. H. MacCullough	

[a]Sample letters follow Table X.

[b]Source: List of manufacturers and consultants taken from index of advertisers in Industrial Quality Control, convention issue, May 1950, page 66, and additional organizations represented by the National Officers of the A. S. Q. C.

[c]Source: List of schools from "Engineering colleges reporting undergraduate and graduate courses in quality control, year 1949-1950," Industrial Quality Control, Vol. VI, No. 4, January 1950, page 29. Individuals teaching S.Q.C. were addressed by name, title and department, as found in the latest catalogues available in office of Registrar, Georgia Institute of Technology, July 10, 1950.

TABLE X

ANALYSIS OF RESPONSES TO QUESTIONNAIRE

Equipment	Application	Using Organizations[a]
Beads, Colored	General	4,6,9,27,51,59,62, 66,74,82,83,84,89, 90,97,98,107,108
	p charts	17,74,84
	Sampling w/paddle	68,74,84,95,103,108
	Sampling Machine	74,84
Beads, Special Set	Disprove 100% Sampling	6,68
Beads on Wires	Control charts	6,10,62,98,108
Blocks, Set of	Sum of Tolerances	3,15,33,68,74,82,84, 95,100,104,109
Cards, Playing	General	3,41,82
	Normal distribution of $\overline{X}$	108
Card, Punched	Normal Curve	19
Charts	General	4,17,27,34,66,80,89, 107
Chips, Marked	General	3,23,41,62,68,82,84, 89,90,97,98,104
	Control Charts	68,95
	Characteristics of $\overline{X}$, R and	17,68,83,95,100
	Correlation	66,68

TABLE X

ANALYSIS OF RESPONSES TO QUESTIONNAIRE (CONTINUED)

Equipment	Application	Using Organizations[a]
Coins	General	41,45,67,84,107
Dice	General	3,9,27,41,45,51,62, 66
	Sampling	104
	Chance Variation	17
	Control Charts	19,23
Dice, Phony	Control Charts	23,27,62,68,74,84
Electrical Device	Analysis of Variance	98
Frequensonar	(same as Quincunx)	6
Model (plexi-glass)	$\overline{X}$ Relationships	83
Parts, Manu-factured	$\overline{X}$ and R charts	17,20,25,68,72,77,89, 100,104
Peg Boards w/washers	Distributions	19,67
Quality Control Indicator	C and p Control	29
Quincunx	General	9,19,23,33,46,62,68,80, 95,104,109
	Control Charts	33,46,68,104
Ring and Plug Gages	Tolerances	15

TABLE X

ANALYSIS OF RESPONSES TO QUESTIONNAIRE (CONTINUED)

Equipment	Application	Using Organizations[a]
Roulette Wheels	Probability	41
Slides	General	4,27,84,99
Slot Machines	Probability	74,84
Templates	$\overline{X}$ Relationships	19

[a]See Table IX, Appendix, for reference numbers to identify industries and colleges.

THE UNIVERSITY OF TENNESSEE
KNOXVILLE
Department of Industrial Engineering

July 19, 1950

Addressed to Specific Individuals
Teaching Statistical Quality Control
in Engineering Colleges

This is a request for your advice and suggestions concerning the use of visual aids in the teaching of statistical quality control. Specifically we are trying to assemble the most progressive ideas concerning demonstrations and/or experimental procedures for student participation which can be used to invigorate interest and supplement instruction in elementary statistical quality control and sampling techniques.

If you or other members of your staff have found it profitable to utilize any special apparatus other than dice, coins, or drawing of colored beads to illustrate the testing of statistical theory or its application to one or more phases of quality control or acceptance sampling we will be most grateful for a brief description of what was used and the applications which appeared to be most suitable.

It is our purpose to prepare a series of progressive laboratory demonstrations using apparatus which is inexpensive or readily available in the average industrial plant or engineering college laboratory to more effectively teach various types of control charts and acceptance sampling procedures. It is understood, of course, that appropriate acknowledgement will be given for any occasion to utilize your suggestions.

A self-addressed envelope is enclosed for your convenience.

Very truly yours,

/s/ Robert M. LaForge

Robert M. LaForge
Assistant Professor
Industrial Engineering

RML/err
encl.

240704

THE UNIVERSITY OF TENNESSEE

KNOXVILLE

Department of Industrial Engineering

July 26, 1950

Addressed to the
Director of Statistical Quality Control
in leading industrial organizations

Dear Sir:

This is a request for your advice and suggestions concerning apparatus suitable for demonstrations and/or experimental procedures which can be used to supplement and crystallize instruction in statistical quality control and sampling techniques.

In the course of your work, you probably find it necessary at times to carry on a certain amount of training for new employees in your department, and perhaps for more experienced employees in other departments or in management, to orient them concerning elementary statistical quality control. If in this instruction, you have found it profitable to utilize any special apparatus other than dice or drawing from a bowl of colored beads, we will appreciate a brief description of the equipment, and the applications for which it was used.

It is my purpose to prepare a series of progressive laboratory demonstrations using apparatus which is relatively inexpensive or readily available in the average industrial plant or engineering college which will help develop in students a clearer understanding of various types of control charts and acceptance sampling procedures. If this material is used in a publication appropriate credit will be given, and the publication made available to you.

Please mail your suggestions in the enclosed self-addressed envelope.

Very truly yours,

/s/ Robert M. LaForge

Robert M. LaForge
Assistant Professor
Industrial Engineering

RML/err
encl.